



Seat No. : _____

TH-120

B.C.A. Sem.-I

May-2013

CC-104 : Basics of Mathematics (BM)

Time : 3 Hours]

[Max. Marks : 70

1. (A) (a) $A = \{2x/x \in \mathbb{Z}\}$, $A = \{4x/x \in \mathbb{Z}\}$ and $C = \{8x/x \in \mathbb{Z}\}$ then find, 6
- (i) $(A \cup B)$
- (ii) $(A \cap B)$
- (iii) $(A \cup B) \cap (A \cup C)$
- (b) Let $f(x) = 2^x$, then find $f\left(\frac{2}{3}\right)$, $\frac{f(-1)}{2}$ and $\frac{f(4)-f(2)}{f(3)}$.

OR

- (A) (a) If $P = \{1, 2, 3\}$, $Q = \{3, 4\}$, $R = \{1, 4\}$ then verify,
 $P \times (Q - R) = (P \times Q) - (P \times R)$
- (b) Let $f(x) = x^2 - 3x + 10$ then find $f(x + 1) - f(x - 1)$.
- (B) If $A = \{x/ |x^3 - 2| \leq 25, x \in \mathbb{N}\}$, $B = \{y/ 1 < y < 5, x \in \mathbb{N}\}$ and $C = \{z/ z^4 = 81, x \in \mathbb{N}\}$ then verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. 4

OR

If $A = \{x \in \mathbb{Z} / x^2 < 49\}$, $B = \{x \in \mathbb{N} / 57 > (x + 1)^2\}$ and $C = \{x \in \mathbb{Z} / -3 < x < 11\}$ then verify $(A \cap C) \Delta (A \cap B) = A \cap (B \Delta C)$.

- (C) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then prove that $f\left(\frac{2x}{1+x^2}\right) = 2 \cdot f(x)$. 4

OR

If $\log\left(\frac{x-y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then prove that $x^2 + y^2 = 11xy$, ($x > y > 0$).

2. (A) Define following terms with example. 6
 Scalar Matrix, Symmetric Matrix, Transpose of a Matrix.

OR

- (A) For given matrices $A = \begin{bmatrix} 2 & 2 & 3 \\ -3 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix}$ find

- (a) $A^2 + 2AB + B^2$
- (b) $(A + AB)^T$

(B) Find the rank of matrices, $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$ 4

OR

(B) If $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ then prove that $A^2 = I$. Deduce that $A^{2n} = I$.

(C) Solve the following system using Cramer's Rule. 4

$$x + 2y + 3z = 6$$

$$2x + 4y + z = 7$$

$$3x + 2y + 9z = 14$$

OR

(C) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ then prove that $A^3 - 3A^2 - A + 9I = O$

3. (A) (a) Find the distance between two points (0, 2) and (4, 5). 6

(b) Find the area of a triangle formed by three points (1, 1), (2, 4) and (5, 2).

OR

(A) (a) What will be the value of x if the distance between $(x, 4)$ and $(-8, 2)$ be 10 ?

(b) Show that the points (6, 6), (2, 3) and (4, 7) are the vertices of a right angled triangle.

(B) If a point $P(1, 2)$ divides a line segment joining points $A(-2, -1)$ and B in the ratio 1 : 3 then find the coordinates of point B . 4

OR

(B) (a) Find the angle between given two lines $2x + 3y + 4 = 0$ and $2x + 3y - 3 = 0$.

(b) Find out an equation of a line having y -intercept 2 and slope 3.

(C) Find the equation of a line passing through the intersection of the lines $x - 2y - 2 = 0$ and $2x - 5y + 1 = 0$ and 4

(a) having slope $\frac{1}{2}$

(b) is perpendicular to $3x - 2y + 7 = 0$

(c) is parallel to $2x - 5y + 9 = 0$

(d) having x -intercept is 3.

OR

(C) Find the equation of two lines through the point (2, -1) making an angle of 45° with the line $6x + 5y - 5 = 0$.

4. (A) (a) $\lim_{x \rightarrow 0} \left(1 + \frac{2x}{5}\right)^{\frac{2}{x}}$
 (b) Find $\frac{dy}{dx}$ when $y = 5x^4 - e^x$

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OR

- (A) (a) Check the continuity of $f(x)$ at $x = 5$,
 $f(x) = \frac{x^2 - 25}{x - 5}, x < 5$
 $= 2x - 5, x \geq 5$

(b) $\int \cos^2 x \sin x \, dx$

- (B) (a) Find $\frac{dy}{dx}$ when $y = 3^x e^x + \log x$
 (b) Find $\frac{dy}{dx}$ when $y = e^{2x+3}$

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OR

- (B) (a) Find $\frac{dy}{dx}$ when $y = 100x^2 \log x$
 (b) Find $\frac{dy}{dx}$ when $y = \sin^4 x$

- (C) (a) Evaluate $\int_{-1}^1 (x^2 + 2^x) \, dx$

(b) Evaluate $\int \frac{1}{4x-7} \, dx$

4

OR

- (C) (a) Evaluate $\int \left(x + \frac{1}{x}\right) dx$

(b) Evaluate $\int_0^1 (x^2 + 5) \, dx$

5. Do as Directed :

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- (1) Define complement of a set.
- (2) Give the power set of $A = \{1, 2, 3\}$.
- (3) List the element of the set $A = \{x / |x| < 3, x \in \mathbb{Z}\}$.

- (4) State De' Morgan's laws.
 - (5) Give the Range for the function $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = x$.
 - (6) Let $A_{2 \times 3}$ and $B_{2 \times 3}$ then AB is of order 2×3 (True/False)
 - (7) If $A = O$ (Null Matrix of order 2), then find $A^2 + I_2$.
 - (8) Find the slope of a line $3x + 6y + 9 = 0$.
 - (9) Give an equation of a line passing through $(-3, 0)$ and $(2, 3)$.
 - (10) Give an equation of a line passing through a point $(3, 2)$ and having slope 2.
 - (11) As $x \rightarrow 1$, $x + 3 \rightarrow ?$
 - (12) Is the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x$ continuous at $x = 0$?
 - (13) Differentiate the function $y = x + 2x + 3x + 4x + 5x$ with respect to x .
 - (14) Evaluate the integration of the function $y = x + 2x + 3x$ with respect to x .
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