Seat No. : _______________

AC-117

April-2015

B.Sc., Sem.-VI

STA-307 : STATISTICS
(Distribution Theory – II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions :
(1) **All** questions are of equal marks.
(2) Scientific Calculator is allowed to use.
(3) Statistical table will be supplied on request.

1. (a) State standard Cauchy distribution. Let X be a random variable having standard Cauchy distribution. Obtain the probability density function of X^2 and identify the resulting distribution.

OR

State Laplace distribution with scale parameter λ and location parameter μ. For this distribution show that

$$\mu'_r = \frac{1}{2}\sum_{k=0}^{r}\left[\binom{r}{k}\lambda^k \mu^{r-k} k!\ \{1 + (-1)^k\}\right]$$

Hence deduce mean and variance of the distribution.

(b) For a random variable X let E(X) = m and V(X) = s^2. If Y = log X is normal distributed with mean μ and variance σ^2, prove that

(i) $m = \exp\left(\mu + \frac{\sigma^2}{2}\right)$ and (ii) $1 + \frac{s^2}{m^2} = e^{\sigma^2}$

OR

State two parameters Cauchy distribution. Obtain its characteristic function. Hence deduce that the distribution of $\bar{X}$, mean of n observations from standard Cauchy distribution, is also a standard Cauchy.

2. (a) State bivariate normal distribution. Obtain its moment generating function. Deduce it for standard bivariate normal distribution.

OR

Show that (X, Y) possesses a bivariate normal distribution if and only if every linear combination of X and Y viz, $aX + bY$, $a \neq 0$, $b \neq 0$ is a normal variate.

(b) Let $(X, Y) \sim BN(0, 0, 1, 1, \rho)$. For this distribution obtain the recurrence relation

$\mu_{rs} = (r + s - 1)\, \rho\, \mu_{r-1,s-1} + (r - 1)(s - 1)(1 - \rho^2)\, \mu_{r-2,s-2}$. Hence deduce that $\mu_{rs} = 0$ if $r + s$ is odd.

OR

If the exponent of e in a bivariate normal density is

$$-1352\left[25\,(x-1)^2 - \frac{480}{13}(x-1)(y-2) + 16\,(y-2)^2\right]$$

Obtain : (i) The mean and variance of the conditional distribution of Y given X = 3.

(ii) $P(1 < Y < 3/X = 3)$.

3. (a) State and prove Chebychev's inequality. Hence check whether there exist a variate X with $E(X) = \mu$ and $V(X) = \sigma^2$, for which $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) = 0.6$.

OR

Let X follows uniform $U(-1, 3)$ distribution. Obtain upper bound for the probability $P\left\{|X - 1| \geq \frac{4}{3}\right\}$. Compare your answer with the exact probability.

(b) State weak law of large numbers (WLLN). Let $\{X_n\}$be a sequence of uniform distributed random variables over $(-\beta n^{\lambda}, \beta n^{\lambda})$, $\beta > 0$, $\lambda > 0$.Test whether (WLLN) hold for $\{X_n\}$.

OR

State convergence in probability. Let $\{X_n\}$be a sequence of random variables with $P\left(X_n = \frac{2^{r-n}}{2n}\right) = \binom{n}{r}(1/2)^n$, r = 0, 1, 2, …., n. Show that the sequence $\{X_n\}$ converges in probability to zero.

4. (a) State and prove Lindbery-Levy Central Limit Theorem (CLT).

OR

State Liapounov's form of central limit theorem. Let $\{X_n\}$be a sequence of independent Bernoulli variates with $P(X_n = 1) = p_n = 1 - P(X_n = 0)$. Show that $\{X_n\}$ obeys CLT.

(b) Let $\{X_n\}$be sequence of iid Poisson variates with $E(X_1) = 2$. Find $P(190 < X_1 + X_2 + \ldots + X_{100} < 210)$ using CLT.

OR

Let $\{X_n\}$ be sequence of iid standard normal variates find $P\left(60 < X_1^2 + X_2^2 + \ldots + X_{50}^2 < 80\right)$ using CLT.

5. Answer the following :

(i) State the distribution for which WLLN cannot be apply.

(ii) Let (X, Y) ~ BN $\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Under what condition X and Y will be independent random variables ?

(iii) State Cauchy-Schwartz inequality.

(iv) State convergence in distribution.

(v) State two properties of convergence in probability.

(vi) When CLT and WLLN hold for a sequence $\{X_n\}$?

(vii) Let Y = log X follows normal $N(\mu, \sigma^2)$ distribution. State the distribution of e^y.

(viii) State Bernoulli's Law of large numbers.

(ix) State necessary and sufficient condition for sequence $\{X_n\}$to satisfy the WLLN.

(x) State characteristic function of standard Laplace distribution.

(xi) If X and Y are standard normal variates with correlation coefficient ρ, then state the distribution of

$$\frac{(X^2 - 2\rho XY + Y^2)}{(1 - \rho^2)}.$$

(xii) Let (X, Y) ~ BN (0, 0, 1, 1, ρ). State the distribution of Y/X.

(xiii) Let (X, Y) ~ BN (2, 3, 4, 9, 0.7). What is the prob $P(X < 2)$?

(xiv) State general form of T cheby chev's inequality.
