Seat No. : _______________

AF-112

April-2015

B.Sc., Sem.-VI

MAT-310 : Mathematics
(Graph Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) There are **five** questions.
(2) Each question carries **14** marks.
(3) Draw figure / graph wherever necessary.

1. (a) Define the following term with proper graph :
   (i) Complete Graph
   (ii) Multi-graph
   (iii) Adjacent edges
   (iv) Parallel edges

   OR

   Define the following term with proper graph :
   (i) Simple graph
   (ii) Loop
   (iii) Isomorphic graph
   (iv) n-regular graph

   (b) State and prove "First Theorem of Graph Theory".

   OR

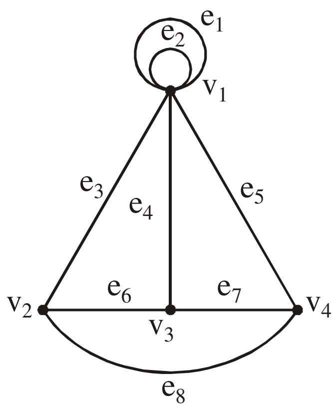
   Let G be a non-empty graph with atleast two vertices, then prove that G is bipartite if and only if it has no odd cycle.

2. (a) Let G be a graph with n vertices $v_1, v_2, \dots v_n$ and let A denote the adjacency matrix of G w.r.t. this listing of the vertices. Let $B = (b_{ij})$ be the matrix $B = A + A^2 + \dots + A^{n-1}$. Then G is connected graph iff B has no zero entries-off the main diagonal.

   OR

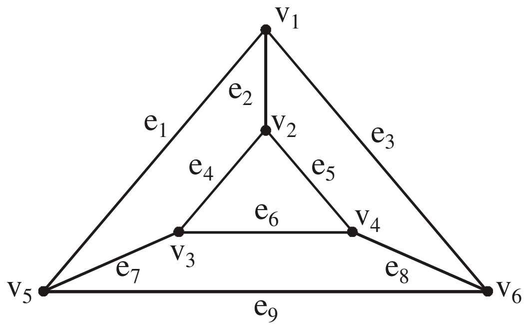
   Let e be an edge of the graph G and G – e be the subgraph obtained by deleting e, then $W(G) \leq G(G - e) \leq W(G) + 1$. (Where W (G) is the number of connected components).

(b) Write down the adjacency matrix and incidence matrix for the following graph :



OR

Write down the adjacency matrix and incidence matrix for the following graph :



3. (a) The complete graph K_n has n^{n-2} different spanning trees.

OR

Let G be simple graph with atleast three vertices then G is 2-connected if and only if for each pair of distinct vertices u and v of G, there are two internally disjoint u-v path in G.

(b) Give seven different spanning trees of K_4.

OR

Let G be a graph with n vertices (where $n \geq 2$), then G has atleast two vertices which are not cut vertices.

4. (a) Discuss Konigsberg bridge problem.

OR

A connected grpah G has an Euler trail if and only if it has atmost two odd vertices.

(b) If G is simple graph with n-vertices (when $n \geq 3$) and the $d(u) \geq \frac{n}{2}$ for every vertex v of G, then prove that G is Hamiltonian.

OR

Discuss "The Travelling Salesman Problem."

5. Answer in short : (Attempt any **seven**)

(i) What is the smallest integer n such that the complete graph K_n has atleast 500 edges ?

(ii) Draw Petersen Graph.

(iii) Give two trees with 7 vertices.

(iv) Let G be a connected with 17 edges then what is the maximum possible number of the vertices in G ?

(v) Discuss whether complete graph K_4 is Euler or not.

(vi) How many different Hamiltonian cycles for complete graph K_5 ?

(vii) Define : Cut vertex with graph.

(viii) Draw self-complementary graph with 4 or 5 vertices.

(ix) Define : "Underlying simple graph" with proper graph.