Seat No. : _____

AG-130

April-2015

B.Sc., Sem.-VI

MAT-311 : Mathematics (A)

(Convex Analysis and Probability Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) Notations are usual everywhere.

(2) Figures to the right indicate marks of the question.

1. (a) Define convex set and affine set. Also give an example of each of them. **9**

OR

Define monotonically increasing and decreasing functions on an interval I. Also show that the function f : R $\rightarrow$ R defined as f (x) = x^2 is monotonically increasing on [0, ∞) and decreasing on ($-\infty$, 0].

(b) If I is an interval and f : I $\rightarrow$ R be a strictly monotonic function such that f (I) is an interval then prove that f^{-} is one-one and continuous. **9**

OR

If I is an interval containing more than one point and f : I $\rightarrow$ R is a differential function such that f ' is nonnegative throughout I then prove that f is monotonically increasing on I.

2. (a) With respect to probability, define random experiment, event, equally likely (or equiprobable) event, complimentary event.

If you roll two dice,

Write the sample space and write the elements of follow events :

A : Two same odd numbers on both dice, B : Two same even numbers on both dice, C : sum of two numbers is less than or equal to 5, D : sum of two numbers is more than 5. Check (i) whether events A and B are mutually exclusive, (ii) whether C and D are mutually exhaustive. Also, write complementary event to D. **9**

OR

State the addition rule of probability for two events and deduce the same for three events.

A student is concerned about his results in Mathematics and Statistics. According to him, probability of passing in Statistics is 0.4, probability of his passing in both the subjects is 0.1 and probability of his passing in atleast one is 0.6. Then, find the probability (i) of his passing in Mathematics, (ii) of passing in Mathematics but not in Statistics, (iii) of passing neither in Mathematics nor in Statistics.

(b) Define mutually exclusive events, mutually exhaustive events, difference events for the following random experiment, verify the results. **9**

$P(\phi) = 0$, $P(\bar{C}) = 1 - P(C)$ $\quad P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Three coins are tossed once. Event defined are :

A = Atleast two heads, B = Exactly two heads are obtained, C = Atmost one head is obtained, D = One head and two tails, Choose null event, ϕ from events defined here.

OR

State classical, axiomatic and conditional probability and Bayes' Rule.

There are two bags, one contains 5 red and 8 black balls and other bag contains 7 red and 10 black balls. A ball is drawn from one or the other bag. Find the chance of drawing a red ball.

3. (a) A manufacturing unit produces door handles. If a random variable X shows the number of defective handles produced in a batch, suggest an appropriate distribution for X and state its probability mass function. If probability of getting a defective door handle is 0.4, then out of 6 defective handles, find probability that (i) 3 defective door handles are chosen, (ii) atmost 2 defective handles are chosen. **9**

OR

State the probability mass function of Poisson distribution. State its mean and variance. Also, state the situations, where Poisson distribution is applied.

If a random variable X has Poisson distribution with mean 2,

find $P(X = 1)$, $P(X < 2)$, $P(1 < X < 4)$

$\{e^{-1} = 0.368, e^{-2} = 0.135, e^{-3} = 0.050\}$

(b) If a random variable X has binomial distribution, with mean 4 and variance (4/3), find P (X = 2), P (X < 2), P (X > 1) **9**

OR

Write a note on Normal distribution. If mean and variance of Normal distribution is zero and one, then state the probability function of normal distribution.

4. Attempt any **eight** of the following questions in short : **16**

(1) Define any **two** of the following terms :

(i) Hyper Plane

(ii) Convex combination

(iii) Convex hull of a set

(2) Explain convex cone with figure.

(3) Give examples each one of convex and non-convex sets of R^2.

(4) If $A = \{(x, y) \in R^2 / x^2 + y^2 \leq 5\}$ then find the convex hull of A.

(5) Two coins are tossed, find the probability that exactly one head appears.

(6) Define certain events and null events with suitable example.

(7) A random variable X follows binomial distribution with parameters (n = 5, p) such that P (X = 1) = P (X = 2), then find p.

(8) For two independent events A, B on a finite sample space S, find the value of $p(\bar{\mathbf{B}} \mid \mathbf{A})$

(9) State two characteristics of normal distribution.

(10) If for a Poisson distribution with parameter m such that 2P (X = 0) + P (X = 2) = 2 P (X = 1), then find the value of m.

Seat No. : _____

AG-130

April-2015

B.Sc., Sem.-VI

MAT-311 : Mathematics (B)

Cryptography (Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) There are **4** questions. **All** questions are compulsory.

(2) Figures to right indicate full marks for the question.

1. (A) State and prove the Fermat's Little Theorem. **9**

OR

Explain following terms with examples :

Ring, Non-commutative ring, Commutative ring, Infinite commutative ring, Additive inverse, Multiplicative inverse, co-prime, discrete logarithm, Linear Diophantine Equation.

(B) Obtain the value of x that satisfies the following four congruence : **9**

$x = 1 \pmod 2$, $x = 2 \pmod 3$, $x = 3 \pmod 5$, $x = 4 \pmod 7$

OR

Evaluate the discrete logarithm of 60 to the base 4 with p = 163.

2. (A) (1) Discuss Modern cryptography.

(2) Discuss relation between Hill cipher and Permutation cipher. **9**

OR

Define Shift cipher and Affine cipher. A cipher-text obtained using the shift cipher is given below. Do the cryptanalysis and obtain the plain text : LKDWHBRXPUVKDQWLODOMXQQXQZDOD

(B) Encrypt the following text using the Hill cipher scheme of

$\begin{bmatrix} 2 & 19 \\ 17 & 14 \end{bmatrix}$: "Cryptography is Cool" **9**

OR

Here is cipher-text that was produced using an affine cipher on the English alphabet with encryption key y = 5x + 4. Find the decryption key and then decrypt the message. "OYHYJLEVYQBLSRIJLYEC".

3. (A) Laila selects p = 17 and c = 3 and convey the same to Majnu. Laila selects a = 15 and Majnu selects b = 13. What is private key exchange between them using the DH algorithm ? Show how Eve mounts an attack using Shank's algorithm and wrenches the private key shared between Laila and Majnu. **9**

OR

Explain the Pollard's ρ algorithm for Discrete logarithm.

(B) Reena and Pragnay select the prime number p = 139 with g = 3 as a primitive elements. Pragnay select a random number k =12 as private key, computes him public key and sends it to Reena ; Reena uses x = 52 as the ephemeral key to mail a message m = 100 to Pragnay. Show the full transaction including the recovery of message key using ElGamal Public-key cryptosystem. **9**

OR

With p = 7, q = 13, e = 5 and m = 10. Show that the complete transaction conforming to the RSA cryptosystem.

4. Do as directed. Attempt any **eight** : **16**

(1) State Euler's theorem and give a simple illustrative example.

(2) A _____ cipher replaces one character with another character.

(3) The _____ cipher is the simplest monoalphabetic cipher. It uses modular arithmetic with a modulus of 26.

(4) What is a 'Monoalphabetic cryptosystem' ? What is a 'Polyalphabetic cryptosystem' ?

(5) What is probability that at least two share a birthday from group of n people ?

(6) A combination of an encryption algorithm and a decryption algorithm is called a _____.

(7) What is the fundamental theorem of arithmetic ? Give an illustrative example.

(8) The _____ is the original message before transformation.

(9) A (n) _____ algorithm transforms plain-text to cipher-text.

(10) Write two properties of primitive element in Z_p.

_____

Seat No. : _____

AG-130

April-2015

B.Sc., Sem.-VI

MAT-311 : Mathematics (C)

Operation Research

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Figures to right indicate full marks of the question.

1. (A) Explain the EOQ (Economic Order Quantity) model with constant rate of demand. **9**

OR

Explain the order level, lot size (OLLS) system.

(B) Find the most economic batch quantity of a product on a machine if the production rate of that item on the machine is 200 pieces per day and the demand is uniform at the rate of 100 pieces per day. The ordering cost is ₹ 200 per batch and the cost of holding one item in inventory is ₹ 0.81 per day. How will the batch quantity vary if the production rate is infinite ? **9**

OR

The demand for a certain item is 16 units per period. Unsatisfied demand causes a shortage cost of ₹ 0.75 per unit per short period. The cost of initialing purchasing action is ₹ 15 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is ₹ 8 per unit. Find the minimum cost and purchase quantity.

2. (A) Explain the basic difference between PERT and CPM. **9**

OR

Explain the terms in brief : (1) Events (2) Activities

(B) Consider the following project activity and their duration. **9**

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
Immediate Predecessor	E	A	B	K	-	E	F	F	F	I,L	C,G,H	D	I,L
Duration	4	2	1	12	14	2	3	2	4	3	4	2	2

Find :

(i) The network diagram showing the inter-relations between the various elements of project.

(ii) The minimum time those take to complete the project.

OR

Consider the following information on the activities required for project.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Immediate Predecessor	-	-	-	A	A	E	B	B	D,F	C	H,J	G,I,K
Duration	2	2	2	3	4	0	7	6	4	10	3	4

Construct the project network. Find the critical path. Also compute total float and free float for non-critical activities.

3. (A) Explain : (i) Pay-off matrix (ii) Assumption of the game **9**

OR

Explain : the principles of dominance.

(B) Solve the game whose pay-off matrix is given below : **9**

		Player B			
		B_1	B_2	B_3	B_4
	A_1	– 1	2	3	0
Player A	A_2	– 4	– 1	– 1	0
	A_3	– 1	1	1	– 4
	A_4	4	– 1	2	– 7

OR

Solve the following game :

		Player B		
		B_1	B_2	B_3
	A_1	3	1	1
Player A	A_2	1	1	5
	A_3	1	4	1

4. Attempt any **eight** : **16**

(1) Define : Inventory. List the types of inventory.

(2) Write down the EOQ formula for EOQ model with finite replenishment rate.

(3) In the EOQ model with constant demand, if the order quantity decreased by 25% then how much total cost decrease ?

(4) Define : Lead time, Cycle time.

(5) Draw the network for the following information :

Activity	A	B	C	D	E	F
Immediate Predecessor	-	-	-	A,B	B,C	D,E

(6) Explain : Dummy activity.

(7) List any two methods which are used to solve the games without saddle point.

(8) Give an example of pay-off matrix for game without saddle point.

(9) Determine the value of the game with the pay-off matrix.

		Player B		
		B_1	B_2	B_3
Player A	A_1	3	2	– 2
	A_2	1	– 3	– 4
	A_3	0	1	– 3

(10) Define : A fair game with illustration.

———————