Seat No. : _______________

BC-101

May-2015

B.Sc., Sem.-IV

CC-204 : Statistics
Random Variable and Probability Distribution-II

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) Attempt **all** questions.

(2) Each carry equal marks.

(3) Scientific calculator are allowed.

1. (a) State and prove inversion theorem on characteristic function.

OR

If characteristic function of a random variable X

$\phi_x(t) = 1 - |t|$ if $|t| < 1$

$= 0$ if $|t| \geq 1$

then find the probability function of X.

(b) State and prove Boole's inequality

OR

$X \sim N(\mu, \sigma^2)$ then find the characteristic function of X and also find mean.

2. (a) If X and Y are two independent variates such that $X \sim G(m)$, $Y \sim G(n)$ then derive the distribution of $\frac{x}{x+y}$.

OR

Define Weibull distribution with three parameters and obtain its mean, mode and variance.

(b) For a normal distribution find mean, median and mode.

OR

Define normal distribution and obtain m.g.f. and hence find mean and variance.

3. (a) For any two random variables X and Y prove that $V(X) = E[V(X|Y)] + V[E(X|Y)]$.

OR

The expected value of X is equal to the expectation of the conditional expectation of X given Y = y. And also state the properties of conditional mean.

(b) The joint p.d.f of two random variable X and Y is

$f(x,y) = \frac{1}{3}(x+y)$; $0 \leq x \leq 2$, $0 \leq y \leq 1$, find the marginal distributions of X and Y and conditional distributions of X given y and Y given x.

OR

The joint density function of X and Y be given by

$$f(x,y) = y_0 x^2 y^3, \; 0 < x < y < 1$$
$$= 0 \quad , \text{ o.w}$$

Find y_0 and conditional mean and variance of X given Y = y.

4. (a) Explain Markov chain, also define transition probability matrix. Also state the properties of transition probability matrix.

OR

State and prove Chapman-Kolomogrov equation.

(b) The pattern of sunny and rainy days on the planet Rainbow is homogeneous Markov chain with two states. Every sunny day is followed by another sunny day with probability 0.8. Every rainy day is followed by another rainy day with probability 0.6.

(1) Today is sunny on rainbow. What is the chance of rainy the day after tomorrow ?

(2) Compute the probability April 1 next year is rainy on Rainbow.

OR

A 3-state Markov chain $\{X_n, n = 0, 1..\}$ has the transition probability matrix

$$P = \begin{pmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{pmatrix}$$

(1) Compute the two-step transition matrix

(2) What is $pr\{X_3 = 1 \mid X_1 = 0\}$

(3) What is $pr\{X_3 = 1 \mid X_0 = 0\}$

5. Answer the following :

(1) If X and Y are independent standard normal variates then write mean and variance of (X – Y).

(2) Write pdf and mode of Weibull distribution with two parameters.

(3) State any two properties of characteristic function.

(4) What is the effect of change of origin and scale on characteristic function ?

(5) State the condition when Markov chain becomes doubly stochastic Markov chain.

(6) Define : Bivariate product moment.

(7) Define : Independent random variables.