Seat No. : _______________

AJ-104

April-2015

M.Sc., Sem.-IV

MAT-510 : Mathematics

Quantitative Techniques

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**

   (i) Define negative binomial distribution and find its first moment.

   (ii) Find r-th moment of exponential and gamma distributions.

   (b) Attempt any **one** : **4**

   (i) Prove that Poisson distribution is limiting case of binomial distribution.

   (ii) The mean and variance of binomial distribution are 8 and 4/3 respectively. Discuss the existence of experiment.

   (c) Attempt any **one** : **3**

   (i) The inner diameter of a cylinder has the specifications 1 ± 0.03 in. The matching process output follows a normal distribution with the mean 1 cm and standard deviation 0.1 cm. Determine the percentage of production that will meet the specifications. Use $P(z \leq 0.3) = 0.6179$.

   (ii) What is the need of distributions ? Explain with example.

2. (a) Attempt any **one** : **7**

   (i) Give formula of optimum order quantity (Define each symbol used) and determine :

   (a) Optimum number of orders placed per year

   (b) Optimum cycle time

   (c) Optimum total cost

   (d) Average inventory

   (ii) Discuss EOQ problem with floor constraint.

   Consider a shop which stores three items. The demand rate for each item is constant and can be assumed to be deterministic. Shortages are not allowed. The relevant data for the items is given in the following table :

Item	A	B	C
Demand rate (unit / year)	2	4	4
Holding cost (₹)	0.30	0.10	0.20
Set-up cost per lot (₹)	10	5	15
Floor space required (ft^2.)	1	1	1

   Determine the optimum lot size for each item when maximum allowable storage area is 25 ft^2.

(b) Attempt any **one** : **4**

(i) What are the types of inventories ?

(ii) Discuss the various factors affecting inventory control.

(c) Fill in the blanks : **3**

(i) The stock of materials kept in the stores in the anticipation of future demand is known as _____.

(ii) Insurance charges of materials cost fall under the inventory _____ cost.

(iii) If the total investment is limited, then the best order quantity for each item will be _____ the EOQ.

3. (a) Attempt any **one** : **7**

(i) Prove that death follows Poisson distribution.

(ii) Derive expected number of customers in the system and the queue for ((M / M / 1) : (1 / FCFS)) queue.

(b) Attempt any **one** : **4**

(i) A tollgate is operated on a NH-8 where vehicles arrive according to Poisson distribution with a mean frequency of 1.2 vehicles per minute. The time of completing toll tax payment is an exponential distribution with mean 20 seconds. Find :

(a) the probability that a vehicle spends more than 30 seconds in the system and

(b) the average waiting time in the queue for those who wait.

(ii) In a doctor's clinic, patients arrive according to Poisson distribution at an average rate of 15 patients / hour. The waiting room can accommodate 14 patients. The examining time per patient is exponentially distributed with the mean rate of 10 / hour. Find :

(a) effective arrival rate of patients in the clinic and

(b) expected waiting time of a patient in the clinic.

(c) Define any **three** : **3**

(i) Queue (ii) Balking (iii) Queue discipline (iv) Capacity of source

4. (a) Attempt any **one** : **7**

(i) Derive the replacement policy when the value of money does not change with time.

(ii) At the starting time when t = 0, all items in a system are new. Each item has a probability of failing immediately before the end of the first month of life and a probability q = 1 – p of failing immediately before the end of the second month. If all items are replaced as they fail, show that the expected number of failures f(x) at the end of month x is given by $f(x) = \frac{N}{1 + q} \left[1 - (-q)^{x+1}\right]$ where N is the number of items in the system.

(b) Attempt any **one** : **4**

(i) The cost of a machine is ₹ 60,000. The data found from experience is as follows :

Year	**1**	**2**	**3**	**4**	**5**
Resale value (₹)	42,000	30,000	20,400	14,400	9650
Cost of spares (₹)	4000	4270	4880	5700	6800
Cost of labour (₹)	14,000	16,000	18,000	21,000	25,000

When should the machine be replaced ?

(ii) For a LED bulb, the following failure rates have been observed.

Week	**1**	**2**	**3**	**4**	**5**
% failing by end of week	10	25	50	80	100

There are 1000 bulbs in use and it costs ₹ 2 to replace an individual bulb that has burnt out. If all the bulbs were replaced simultaneously, it would cost ₹ 0.50 per bulb. It is proposed to replace all the bulbs at fixed intervals, whether or not they have burnt out or not and to continue replacing burnt out bulbs as they fail. When should all the bulbs be replaced ?

(c) Choose the correct answer : **3**

(i) The group replacement policy is suitable for similar low cost items, when

(a) the failure is sudden.

(b) the failure is complete and sudden.

(c) the failure occurs over a period of time.

(d) none of the above.

(ii) The problem of replacement is faced when a working item fails

(a) gradually

(b) suddenly

(c) both gradually and suddenly

(d) none of the above

(iii) The present worth factor of one rupee sent in n years' time from now onwards is

(a) $(1 + r)^n$ (b) $(1 + r)^{-n}$ (c) $(1 - r)^{-n}$ (d) none of these

5. (a) Attempt any **one** :

(i) Haggins plumbing and heating maintains a stock of 30 – gallon hot water heaters that it sells to home-owners and installs for them. The owner likes the idea of having large supply on hand so as to meet all customer demand, but he also recognizes that it is expansive to do so. He examines hot water heater sales over the past 50 weeks and the data is **7**

Hot water heater sales per week	**Number of weeks this number was sold**
4	6
5	5
6	9
7	12
8	8
9	7
10	3

Given the following random numbers :

10 24 03 32 23 59 95 34 34 51 08 48 66 97 03 96 46 74 77 44

Compute :

(i) If Higgins maintains a constant supply of 8 hot water heater in any given week, how many times will he be out of stock during the 20-week simulation period ?

(ii) What is the average number of heaters demanded per week over the 20-week interval ?

(ii) A doctor schedules all his patients for 20-minutes appointments some of the patients take more or less than 20 minutes depending on the type of the treatment required. The following table gives the probabilities and time actually require to complete the treatment :

Treatment	**Time required (minutes)**	**Probability**
A	25	0.20
B	10	0.35
C	20	0.25
D	30	0.20

Simulate the doctor's appointments for 2 hours and determine the average waiting time for the patients as well as the idleness of the doctors. Assume that the patients are very punctual. The doctor starts at 10.00 am. Use the random numbers : 40 82 11 30 27 75 32 56.

(b) Attempt any **one** : **4**

(i) State the major reasons for using simulation. Explain the basic steps involved in Monte Carlo simulation.

(ii) What are the advantages and disadvantages of simulation ?

(c) Fill up the blanks : **3**

(i) _____ is not an analytic method.

(ii) While assigning random numbers in Monte Carlo simulation, it is necessary to develop a _____ probability distribution.

(iii) The step required for simulation in solving a problem is to _____ and _____ the experiment.