Seat No. : _______________

BG-118

May-2015

M.Sc., Sem.-II

407 : Statistics

(Reliability and Life Testing and Bayes Estimation)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are of equal marks.

(2) Scientific calculator is allowed.

(3) Statistical Tables will be provided on request.

1. (a) Define hazard function. Describe different types of hazard rates with illustrations. In usual notations prove that $R(t) = \exp\left[-\int_0^t h(x)dx\right]$.

OR

(a) A random sample with observations $X_1, X_2, \ldots, X_n$ be taken from the cdf F(x). Let $Y = \underset{1 \le i \le n}{Min} X_i$ and $U = \underset{1 \le i \le n}{Max} X_i$ then obtain expression for hazard function of Y and U in terms of the hazard function of X.

(b) State mtbf and mttf. Distinguish difference between them. In usual notations prove that $\text{mtbf} = \int_0^\infty R(t)dt$.

OR

(b) Discuss the nonparametric estimation method for estimating hazard rate, reliability and pdf at time t from the data of ungrouped life times of n items.

2. (a) Obtain expressions for reliability for series and parallel systems each having n components having iid life times. What do you conclude from your results ?

OR

(a) Show that the expected life of the series system having two independent components is less than that of the either component, but in case of parallel system it is more than that of either component.

(b) Consider a device in which a component is replaced immediately upon failure by another component of the same type. The lifetime of the component is exponential with mean θ. Replacement of the part can be stocked once a month. Suppose that $\theta = 0.2$ months, how many spares should be stocked so that the probability of shortage will not exceed 0.05 ?

OR

(b) 10 components having iid exponential life time with mean failure time 5 per hour are put on a test without replacement. Find (i) probability that time between fifth and sixth failures is at least 2 hours, (ii) expected time between sixth and seventh failures, (iii) expected time of sixth failure.

3. (a) Distinguish between Type-I and Type-II censoring and point out their uses in life testing experiments. Construct general form of the likelihood function for both the types of censoring schemes under without replacement.

OR

(a) Consider exponential life time model with mean $\theta > 0$. If n units having such life time distribution are placed on the life test, obtain MLE and UMVUE of reliability at time t under Type-II censoring without replacement.

(b) State Weibull life time model. Obtain MLE of the parameters of the model under Type-I censoring without replacement.

OR

(b) Discuss Sinha and Fu (1977) method of estimating failure rate. Describe how do you use this method to obtain estimate of the parameters of the Weibull life time model.

4. (a) Distinguish the difference between the classical method of estimation and Bayesian method of estimation. Define prior and posterior distributions used in the Bayes estimation.

OR

(a) Define risk function, Bayes risk, Bayes estimator. Obtain extensive rule to obtain Bayes estimator.

(b) Under squared error loss and weighted squared error loss functions obtain the general form of the Bayes estimator of $\Psi(\theta)$, a function of parameter θ.

OR

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli distribution with mean θ, $0 < \theta < 1$. Let prior distribution for θ is bita type one $\beta(a,b)$, $a < b$. Obtain Bayes estimator of $1/(\theta(1-\theta))$ under (i) squared error loss function and (ii) weighted squared error loss function with weight $w(\theta) = \sqrt{\theta}$.

5. (a) Answer shortly : (Any **Five**)

(i) Define reliability

(ii) If hazard function of a component is $h(t) = t/\sigma^2$, obtain corresponding life time distribution.

(iii) Let 0.40, 0.55, 0.75, 0.80, 1.05, 1.10, 1.20 be the ordered life times of 7 components, obtain the estimate of reliability at time 0.80.

(iv) For the pdf $f(x) = (a + bx) \exp\left[-\left(ax + \frac{bx^2}{2}\right)\right]$, $x > 0$ obtain reliability at time t.

(v) For exponential life time model reliability at time t = 10 hrs is 0.5, what is the reliability at time 22 hours ?

(vi) For exponential life time model with mean $\theta = 1$,

let $Y = \frac{\sum_{i=1}^{r} X_{(i)} + (n-r) X_{(r)}}{r}$ denote the total test time under Type-II censoring without replacement. If r = 6, calculate $P(Y > 7.906)$.

(b) Find correct answer from the given options.

(i) For exponential life time model under Type-II censoring expected termination time under without replacement is ______ expected termination time under with replacement.

(a) $>$ (b) $<$

(c) $=$ (d) not comparable with

(ii) For exponential life time model with mean θ, under type II censoring with replacement scheme, MLE of θ is

(a) $\frac{\sum_{i=1}^{r} X_{(i)} + (n-r) X_{(r)}}{n}$ (b) $\frac{\sum_{i=1}^{r} X_{(i)} + (n-r) X_{(r)}}{r}$

(c) $\frac{rX_{(r)}}{n}$ (d) $\frac{nX_{(r)}}{r}$

(iii) In Bayes estimation the most appropriate prior distribution for parameter θ, of Rayleigh distribution is

(a) Gamma (a, b)

(b) Bita Type-I $\beta(a, b)$

(c) Normal N(a, b)

(d) Uniform U(a, b)

(iv) For exponential distribution with mean $1/\theta$, $\theta > 0$, consider an estimator T $= \bar{x}$ for the mean, the risk function of T under squared error loss function is

(a) $\frac{\theta^2}{n}$

(b) $\frac{1}{n\theta^2}$

(c) $\frac{n-1}{\theta^2}$

(d) $\frac{\theta^2}{n-1}$