SEMESTER I

MAT401 – Functions of Several Variables – I

Objectives and outcome:
This course is intended to cover two important theorems of mathematics viz. (i) The Inverse Function Theorem (ii) The Implicit Function Theorem. It also prepares proper background for courses in Differential Geometry which are to follow.

Unit I : Euclidean Space, Linear functions, convex sets, Convex and Concave functions, non-Euclidean norms.

Unit II : Directional and Partial Derivatives, Differentiable functions of class $C^{(q)}$, more on convex and concave functions, relative extrema, differential 1-forms.

Unit III : Derivatives, Curves in Euclidean Spaces, Line Integrals, Gradient Method.

Unit IV : Transformations, Linear and affine transformations, Differentiable transformations, Composition, the inverse function theorem.

Unit V : The implicit function theorem, Manifolds, the multiplier rule.

Text book:
Chapters 1 to 4: 'Functions of Several Variables'
- Wendell H. Fleming, Addison-Wesley.

Reference books:

1. 'Calculus of Several Variables', Casper Goffmann, Harper and Row.
2. 'Calculus on Manifolds', Michael Spivak, W. A. Benjamin Inc. N.Y.

MAT402 – Metric Spaces

Objectives and outcome:

The objective of this course is to give a very streamlined development of a course in metric space topology emphasizing only the most useful concepts, concrete spaces and geometric ideas, which are useful in almost all courses of mathematics.

Unit I : Metric and Metric Spaces, Metric from an inner product and a norm, Open balls and Open sets, Equivalent metrics, Interior of a set, Subspace topology. Omit proofs of Cauchy-Schwarz, Young's, Hölder's and Minkowski Inequalities.

Unit II : Convergence of a sequence, Limit and Cluster points, Bolzano Weierstrass Theorem, Cauchy sequences and Completeness, Bounded sets, Dense sets, Basis, Boundary of a set.

Unit III : Continuous functions, Equivalent definition of continuity, Distance between two sets, Urysohn's Lemma for metric space, Gluing Lemma, Topological property, Uniform continuity, Limit of a function, Open and closed maps.

Unit IV : Compact spaces and their properties, Continuous functions on Compact spaces, Characterization of compact metric spaces.

Unit V : Connected spaces, Product of two connected spaces, Path connected spaces.

Text book:

Chapter 1 to 5 from "Topology of Metric Spaces"
- S. Kumaresan, Narosa Publishing House, 2005.

Reference books:

(1) "Introduction to Real Analysis"
- R. G. Bartle and D. R. Sherbert
(3rd edition), John Wiley & Sons (ASIA), 2000

(2) "Principles of Mathematical Analysis"
- Walter Rudin
(3rd edition), International Student Edition, McGraw-Hill, 1985.

(3) "Various Articles on Topology"
- S. Kumaresan
available at http://math.mu.ac.in/mtts/downloads.html/

MAT403 – Complex Analysis-I

Objectives and outcome:

The main aim is to make students familiar with complex numbers, their properties and the study of functions of a complex variable.

Unit I	**:**	Definitions and notations, Algebraic properties, Polar coordinates and Euler's formula, Products and Quotients in exponential form.
Unit II	**:**	Continuous complex functions, Differentiable complex functions, Cauchy Riemann equations.
Unit III	**:**	Reflection principle, Harmonic functions of two variables, Elementary functions.
Unit IV	**:**	Contours, Contour integrals, Anti-derivatives, Cauchy Goursat theorem, Simply connected domain.
Unit V	**:**	Cauchy's integral formula, Derivatives of analytic functions

Text book:

The course is roughly covered by

"Complex variables and applications" (6^{th} edition)

- J. W. Brown and R. V. Churchill, MacGraw Hills.

Reference books:

(1) "Introduction to Functions of Complex Variable"
- C. J. Hamilton, Marcel Dekker Inc. Newyork.

(2) "Complex Analysis"
- I. Stewart and David Tall, Cambridge University Press.

(3) "Complex Analysis"
- J. C. Duncan, John Wiley & Sons, London.

MAT404 – Ordinary Differential Equations

Objectives and outcome:
The objective of this course is to continue the study of ordinary differential equations begun in B. Sc., with an emphasis on second degree equations which occur in applications.

Unit I	**:**	Review of second order linear equations. Series solutions of first order equations. Second order linear equations: ordinary points.
Unit II	**:**	Second order linear equations: regular singular points; Gauss's hypergeometric equation; the point at infinity.
Unit III	**:**	Hermite polynomials. Chebyshev polynomials and the minimax property. Legendre polynomials; properties of Legendre polynomials.
Unit IV	**:**	Bessel functions; properties of Bessel functions; Bessel's integral formula.
Unit V	**:**	Existence and uniqueness of solutions: the method of successive approximations; Picard's theorem; systems of equations.

Text book:

"Differential Equations with Applications and Historical Notes" (2nd Edition)
- G. F. Simmons, Tata McGraw-Hill Publishing Co. Ltd., 2008.

Chapter 5 (Omit Appendices C and E),
Chapter 8 (Omit Appendices A and B),
Chapter 13.

Reference books:

(1) "Introduction to Ordinary Differential Equations"
- A. L. Rabenstein, Academic Press.

(2) "Advanced Engineering Mathematics" (8th Edition)
- Erwin Kreyszig, Wiley-India, 2008.

MAT405 – Measure and Integration

Objectives and outcome:

The initial objective of the course is to introduce the concept of Lebesgue measure for bounded subsets of $\mathbb{R}$. This concept of Lebesgue measure is later used in developing the theory of (Lebesgue) integration which gives stronger (and better) results as compared to the theory of Riemann integration.

Unit I : The structure of open sets in $\mathbb{R}$, Length of open sets and closed sets, Inner and outer measure of bounded sets, Measurable sets and some of its properties.

Unit II : Further properties of measurable sets, Non-measurable sets, Definition and the properties of Measurable functions.

Unit III : A quick review of the definition of Riemann integral, Lebesgue integral for bounded functions and its comparison with Riemann integral, properties of Lebesgue integral for bounded functions.

Unit IV : The Lebesgue integral of non-negative and unbounded functions, its properties, Lebesgue dominated convergence theorem, Fatou's Lemma and its consequences like Monotone convergence theorem and the countable additivity of the Lebesgue integral, Lebesgue integral on $(-\infty, \infty)$.

Unit V : Square integrable functions, the Schwarz and Minkowski's inequality, Completeness of $L^2[a,b]$, Dense sets in $L^2[a,b]$, definition and introduction to Fourier series of integrable functions.

Text book: The course is based on the book
"Methods of Real Analysis"
-Richard Goldberg – Oxford & IBH Publishing Company, 1964.

Unit-I	Chapter - V	(Theorem 5.4 F)
	Chapter – XI	(Section - 11.1 & 11.2)
Unit-II	Chapter – XI	(Section - 11.3 & 11.4)
Unit-III	Chapter – VII	(Section - 7.1 & 7.2)
	Chapter – XI	(Section - 11.5 & 11.6)
Unit-IV	Chapter – XI	(Section - 11.7 & 11.8, 11.10A, 11.10B, 11.10C)
Unit-V	Chapter – XI	(Section - 11.9)
	Chapter – XII	(Section - 12.1)

Reference books:

(1) "Theory of Functions of a real variable" Volume-I
- I. P. Natanson, Frederic Ungar Publishing Co., New York 1964.

(2) "Real Analysis"
- H. L. Royden (3rd edition), Pearson Prentice Hall (2007).

(3) "Measure and Integration"
- I. K. Rana, Narosa Publishing House (1997).

MAT406S – Seminar

The course for seminar is based on the first five papers (MAT 401 to MAT 405).

SEMESTER II

MAT407 – Differential Geometry – I

Objectives and outcome:
This course prepares background for the Differential Geometry - II course. It covers the well-known Frenet Formula, gives ample scope and leisure for building up Geometry Intuition by incorporating classical curves, and related results, along with the course.

Unit I : What is a curve? Arc-Length, Reparametrization, Level curves Vs. Parametrized curves.

Unit II : Curvature, Plane curves, Space Curves, Frenet –Serret Equations.

Unit III : What is a surface? Smooth surfaces, Tangents, Normals & Orientability.

Unit IV : Examples of Surfaces, Quadric Surfaces, Triply Orthogonal Systems, Applications of the Inverse Function Theorem.

Unit V : Lengths of Curves on Surfaces, Isometries of Surfaces, Conformal Mappings of Surfaces, Surface Area, Equiareal Maps & a Theorem of Archimedes.

Text book: "Elementary Differential Geometry " ,Andrew Pressley ,Springer ,2001

Reference books:

1. 'Differential Geometry of curves and surfaces', Manfredo P. do Carmo. Prentice-Hall, Inc. New Jersey.
2. 'Elements of Differential Geometry', R S Millman and G D Parker, Prentice Hall. Inc. New Jersey.
3. 'Differential Geometry of Three Dimensions' C E Weatherburn. ELBS. Low-priced Textbook. Cambridge University Press.
4. 'Elementary Differential Geometry' - Barrett O'Neill, Academic Press. N.Y.

MAT408 – Algebra-I

Objectives and outcome:

The objective of this course is to study group structure and some related applications of groups. It also provides the basis for further studies.

Unit I : Permutation groups, Alternating group of degree n, Group isomorphisms and their properties, Cayley's theorem, Automorphisms, An application of cosets to permutation groups, The rotation group of a cube and a soccer ball.

Unit II : External direct products and their properties, The group of units modulo n as an external direct product, Applications of external direct products to number theory and cryptography, Normal subgroups, Factor groups and their applications.

Unit III : Internal direct products, Group homomorphisms and their properties, Isomorphism theorems, Fundamental theorem of finite abelian groups, The isomorphism classes of abelian groups.

Unit IV : Conjugacy classes, The class equation, Sylow theorems and their applications.

Unit V : Simple groups, Composition factors of a finite group, Nonsimplicity tests, Generalized Cayley theorem, Index theorem, Embedding theorem, The simplicity of A_5, Burnside's theorem and its applications, Group action.

Text book:

"Contemporary Abstract Algebra"
- Joseph A. Gallian, Narosa Publishing House, 1998.

Chapters: 5, 6, 7 (Omit: Properties of cosets, Lagrange's theorem and consequences), 8, 9, 10, 11, 24 (Omit: The probability that two elements commute), 25, 29.

Note: Omit computer exercises.

Reference books:

(1) "Basic Abstract Algebra" (2nd edition)
- P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul, Cambridge University Press, 1995.

(2) "Algebra"
- Michael Artin, PHI Learning Pvt. Ltd., New Delhi, 2009.

(3) "A Course in Algebra"
- E. B. Vinberg, American Mathematical Society, 2003.

(4) "Basic Algebra"
- P. M. Cohn, Springer, 2003.

(5) "Algebra"
- Thomas W. Hungerford, Springer, 2004.

MAT409 - Complex Analysis-II

Objectives and outcome:

The main aim is to make students familiar with the study of functions of a complex variable.

Unit I : Convergence of Taylor series, Laurent series and Uniqueness, Convergence of sequences and series, Uniform and absolute convergence of power series.

Unit II : Residue theorem, Types of isolated singular points, Residues at poles, Zeros and poles of order m, Behavior of f near removable and essential singular points.

Unit III : Liouville's theorem, Fundamental theorem of Algebra, Maximum moduli principle of functions.

Unit IV : Evaluation of improper integrals involving Sines and Cosines, Definite integrals involving Sines and Cosines.

Unit V : Indented path, Integration along a branch cut, Argument principle, Rouche's theorem, Bi-linear transformation.

Text book:

The course is roughly covered by

"Complex variables and applications" (6^{th} edition)

- J. W. Brown and R. V. Churchill, MacGraw Hills.

Reference books:

(1) "Introduction to Functions of Complex Variable"
- C. J. Hamilton, Marcel Dekker Inc. Newyork.

(2) "Complex Analysis"
- I. Stewart and David Tall, Cambridge University Press.

(3) "Complex Analysis"
- J. C. Duncan, John Wiley & Sons, London.

MAT410 – Partial Differential Equations

Objectives and outcome:
The objective of this course is to introduce partial differential equations, particularly the second order equations of mathematical physics.

Unit I : Review of curves and surfaces; genesis of first order PDE; classification of integrals; linear equations of the first order; Pfaffian differential equations; compatible systems of first order PDE.

Unit II : Charpit's method; Jacobi's method; integral surfaces through a given curve; quasi-linear equations (characteristic curves and the initial value problem).

Unit III : Non-linear first order PDE (characteristic curves and the initial value problem).
Genesis of second order PDE; classification of second order PDE.
One dimensional wave equation: vibrations of an infinite string; vibrations of a semi-infinite string.

Unit IV : Vibration of a string of finite-length; Riemann's method.
Laplace's equation: boundary value problems; maximum and minimum principles; the Dirichlet problem for a circle, for the upper half plane, for a rectangle.

Unit V : Neumann's problem for the upper half plane and for a circle.
Harnack's theorem; Green's function.
Heat conduction problem: infinite rod; finite rod.
Duhamel's principle.
Families of equipotential surfaces.
Kelvin's inversion theorem.

Text book:

"An Elementary Course in Partial Differential Equations" (2nd Edition)
- T. Amaranath, Narosa Publishing House Pvt. Ltd., 2005.

Reference books:

(1) "Elements of Partial Differential Equations"
- I. Sneddon, McGraw-Hill Kogakusha Ltd.

(2) "Methods of Mathematical Physics" Vol.2
- R. Courant and D. Hilbert, Wiley Eastern Pvt. Ltd., 1975.

MAT411 – Real Analysis

Objectives and outcome:

The main objective of the course is to study the differential properties of functions of finite variation and absolutely continuous functions and characterize the absolutely continuous functions in terms of the indefinite integral of Lebesgue integrable functions.

A part of the course is also devoted to the study of the structure of measurable functions and study of Fourier series of L^1 and L^2 functions.

Unit I : Convergence in measure and the related important results, Approximations of measurable functions by bounded measurable functions and continuous functions.

Unit II : Weierstrass approximation theorems, introduction of $L^p[a,b]$ spaces, Holder and Minkowski's inequality, $L^p[a,b]$ as Banach spaces, Dense subsets of $L^p[a,b]$, Weak convergence in $L^p[a,b]$.

Unit III : Monotonic function and its differentiability (assuming Vitali's covering theorem), functions of finite (bounded) variation on [a,b] and its properties, functions of finite variation on $\mathbb{R}$.

Unit IV : Absolutely continuous functions on [a,b], differential properties of absolutely continuous function, the indefinite Lebesgue integral and the fundamental theorem of calculus.

Unit V : Definition of Fourier series and convergence problem, (C,1) summability of Fourier series, the L^2 theory of Fourier series, Convergence of Fourier series, Orthonormal expansions in $L^2[a,b]$

Text book:

The course is based on the following books

(Units – I to IV)

(1) "Theory of Functions of a real variable" Volume-I

a. I. P. Natanson, Frederic Ungar Publishing Co., New York 1964.

Note: We intend to discuss the results of Unit-I for finite valued functions defined on bounded sets only.

(Unit V)

(2) "Methods of Real Analysis"

- Richard Goldberg, Oxford & IBH Publishing Company.

Unit-I	Chapter – IV	Sections 3 & 4
Unit-II	Chapter – IV	Section 5
	Chapter – VII	Section 6
Unit-III	Chapter – VIII	Sections 1, 2, 3, Section 5(Theorem-1 only), Section 9 (up to Theorem-5 only)
Unit-IV	Chapter – IX	Sections 1, 2, Section 4 (up to Theorem-6), Section 7
Unit-V	Chapter – XII	(Methods in Real Analysis by R. Goldberg) complete.

Reference books:

(1) "Real Analysis"
- H. L. Royden (3rd edition), Pearson Prentice Hall (2007).

(2) "Measure and Integration"
- I. K. Rana, Narosa Publishing House (1997).

(3) "Trignomatric Series"
- A. Zygmund, Cambridge University Press (1968).

(4) "Fourier Series", a modern introduction Vol.1
- R. E. Edwards (Springer)

MAT412S – Seminar

The course for seminar is based on the first five papers of semester II (MAT 407 to MAT 411).

SEMESTER III

MAT501 - Functional analysis I

Objectives and outcome:

To introduce the elementary aspects of Banach Spaces and operators.

Unit I: Review of linear spaces, quotient linear spaces, direct sums of linear subspaces, basis of a linear space - existence using Zorn's lemma, linear transformations from a linear space to another, projections on a linear space.

Unit II: Normed linear spaces, Banach spaces, quotient of a normed linear space by a closed linear subspace, continuous linear transformations from a normed linear space to a normed linear space, finite dimensional normed linear spaces.

Unit III: Conjugate space of a normed linear space, Hahn-Banach theorem with consequences, the natural imbedding of a normed linear space in its second conjugate space.

Unit IV: Reflexive spaces, open mapping theorem, projections on a Banach space, closed graph theorem, uniform boundedness theorem, conjugate of an operator on a Banach space.

Unit V: Hilbert spaces, orthogonal complements, complete orthonormal sets in a Hilbert space .

Text book: Introduction to topology and modern analysis by G. F. Simmons, McGraw - Hill Book Co.1963; Chapter 8 (42 onwards) to Ch.10 (upto 54).

Reference books:

(1) "Functional analysis " by B. V. Limaye, New Age International Limited publishers.

(2) " An introduction to Hilbert Spaces" by S. K. Berberian, Oxford Uni. Press, 1959. D.Van Nostrand Co. Inc. Princeton, N.J., 1967, also available in paperback, edition by Springer.

(3) "A Hilbert space problem book" by P. R. Halmos.

(4) "An introductory functional analysis with application "by E.Kreyszig, WSE edition, 1989, paperback.

(5) "Linear Analysis" An Introductory course by Bela Bollobas, Foundation Books, Delhi (Cheap edition), 1994.

MAT502 – Algebra-II

Objectives and outcome:

The objective of this course is to study Ring theory and Field theory, and introduce Galois theory with some applications.

Unit I: Ideals, Factor rings, Prime ideals, Maximal ideals, Ring homomorphisms and their properties, Ring isomorphisms, The field of quotients, Polynomial rings, The division algorithm and consequences, Principal ideal domains.

Unit II: Factorization of polynomials, Unique factorization in Z[x], Irreducible and prime elements in integral domain, Unique factorization domains, Euclidean domains.

Unit III: Extension fields, Splitting fields, Zeros of an irreducible polynomial, Characterization of extensions, Types of extensions, Properties of algebraic extensions.

Unit IV: Finite fields, Geometric constructions, Fundamental theorem of Galois theory.

Unit V: Solvability of polynomials by radicals, Insolvability of a quintic, Cyclotomic polynomials, Cyclotomic extensions, The constructible regular n-gons.

Text book:

"Contemporary Abstract Algebra"
- Joseph A. Gallian, Narosa Publishing House, 1998, fourth edition.

Chapters: 14, 15, 16, 17, 18, 20, 21, 22, 23, 32 and 33.

Reference books:

(1) "Basic Abstract Algebra" (2nd edition)
- P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul, Cambridge University Press, 1995.

(2) "Algebra"
- Michael Artin, PHI Learning Pvt. Ltd., New Delhi, 2009.

(3) "A Course in Algebra"
- E. B. Vinberg, American Mathematical Society, 2003.

(4) "Basic Algebra"
- P. M. Cohn, Springer, 2003.

(5) "Algebra"
- Thomas W. Hungerford, Springer, 2004.

MAT503EA - Number Theory

Objectives and outcome:

To make students familiar and friendly with the properties of positive integers. On this line, we cover some special structures (of numbers) such as: Euclidean domains, unique factorization domains.

Unit I: (Divisibility): Foundations, Division algorithm, greatest common divisor, Euclid's algorithm, Fundamental Theorem, Properties of primes.

Unit II: (Arithmetical Functions): The function [x], multiplicative functions, Euler's (totient) function $\varphi(n)$, The Mobius function $\mu(n)$, The functions $\tau(n)$ and $\sigma(n)$, Brief introduction of convolution of arithmetical functions, perfect numbers.

Unit III: (Congruences): Definitions, Chinese-Remainder theorem, The theorem of Fermat and Euler, Wilson's theorem, Lagrange's theorem, Primitive roots, indices.

Unit IV: (Miscellaneous Topics): Finite, infinite continued Fractions, linear Diophantine equations ax+by=c, Pell's equations, Pythagorean triples, brief introduction of Fermat's last theorem.

Unit V: (Quadratic Fields): Algebraic number fields, the quadratic fields, units, Primes and Factorization, Euclidean fields, The Gaussian field, Gaussian primes.

Text book: A Concise introduction to the Theory of Numbers by Alan Baker (Cambridge Uni. Press, Cambridge).

Reference books:

(1) "An introduction to the Theory of Numbers (5^{th} edition) by Ivan Nivan, H. S. Zuckerman, H.L.Montgomery (John Wiley & Sons Inc.)

(2) Elementary Number theory (Sixth edition) by David M. Burton, Tata McGraw-Hill Publishing Co. Ltd., New Delhi.

MAT503EB – Problem Solving-I

Objectives and outcome:

The objective of this paper is to develop and enhance the problem solving skills of the students. The focus will be on using the theory results skillfully to solve the mathematical exercises. The students opting for this paper are expected to have good understanding of Mathematics.

Unit I: **Problems from:** Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence.

Unit II: **Problems from:** Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.

Unit III: **Problems from:** Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function. Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Unit IV: **Problems from:** Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Unit V: **Problems from:** Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Reference:

Model question papers available at http://csirhrdg.res.in/csirnetmqs.htm

MAT504 - Mathematical Programming

Objectives and Outcome:

The objective of this course is to explore various mathematical programming algorithms to solve real life problems.

Unit 1: Modelling with linear programming

Unit 2: Simplex method, Dual linear programming, Integer programming

Unit 3: Transportation and Assignment problems

Unit 4: Non-linear programming, Lagrangian method, Kuhn-Tucker conditions, Quadratic programming

Unit 5: Dynamic programming, Fractional programming

Text book:

Syllabus is roughly covered by "OPERATIONS RESEARCH" by Nita H. Shah, Ravi M. Gor and Hardik Soni, PHI Publications, New Delhi, 2007.

Reference books:

1. OPERATIONS RESEARCH: AN INTRODUCTION (Eighth edition) by H. A. Taha, PHI Publications, New Delhi, 2006.
2. INTRODUCTION TO OPERATIONS RESEARCH by F. S. Hiller and G. J. Liberman, McGraw Hills Publication.
3. MATHEMATICAL METHODS OF OPERATIONS RESEARCH by T. L. Saaty, Dover Phoenix editions Publications.

MAT505 – Functions of Several Variables-II

Objectives and outcome:

This course is intended to cover Stokes' Theorem which extends several results of Advanced Calculus.

Unit I: Intervals, Measure, integrals over n-dimensional Euclidean space, integrals over bounded sets, iterated integrals, the unbounded case, change of measure under affine transformations, transformation of integrals, coordinate systems in Euclidean spaces of n-dimension, convergence theorems, differentiation under the integral sign, the L^p-spaces.

The proofs of the results which are analogous to the results covered in the earlier courses (MAT405, MAT411) should be omitted.

Unit II: Alternating multilinear functions, multicovectors, multivectors.

Unit III: Induced linear transformations, differential forms, the adjoint and codifferential, special results for n=3.

Unit IV: Regular transformations, coordinate systems on manifolds, measure and integration on manifolds, orientations, integrals of r-forms.

Unit V: The divergence theorem, Stokes' Formula, closed and exact differential forms.

Text book:

Chapters 5 to 7: "Functions of Several Variables"
- Wendell H. Fleming, Addison-Wesley, 1965.

Reference books:

(1) 'Calculus of Several Variables'
- Casper Goffmann, Harper and Row.

(2) 'Calculus on Manifolds'
- Michael Spivak, W. A. Benjamin Inc. N.Y.

MAT506S- Seminar

The Course for seminar is based on the first five papers (MAT501 to MAT505)

SEMESTER IV

MAT507 - Functional analysis II

Objectives and outcome:

To analyse concepts related to operators on Hilbert spaces and Banach spaces.

Unit I: Conjugate space of a Hilbert space, Adjoint of an operator on a Hilbert space, Self-adjoint operators.

Unit II: Normal & unitary operators, projections on a Hilbert space, invariant subspaces, spectrum of an operator – eigenvalues, Spectral resolution of an operator on a finite dimensional Hilbert space.

Unit III: Review of matrices in relation to linear transformations, review of determinants, the spectral theorem for finite dimensional Hilbert spaces and its consequences.

Unit IV: Classification of Spectrum of an operator on a Banach space, Spectrum of shift operator, multiplication operator etc. Gelfand Mazur theorem, Spectral radius formula.

Unit V: Compact operators, Spectrum of a compact operator.

Text book:

1. Introduction to topology and modern analysis by G. F. Simmons, McGraw - Hill Book Co. 1963; Ch.10 (55 onwards) and Ch.11.
2. "Functional analysis" by B. V. Limaye, New Age International Limited publishers, 1996 (Second edition Ch.3(12), Ch.5 (17, 18).

References:

(1) "An introduction to Hilbert Spaces" by S. K. Berberian, Oxford Uni. Press, 1959 D. Van Nostrand Co. Inc. Princeton, N.J., 1967, also available in paperback, edition by Springer.

(2) "A Hilbert space problem book" by P. R. Halmos.

(3) "An introductory functional analysis with application"by E. Kreyszig, WSE edition, 1989, paperback.

(4) "Linear Analysis" An Introductory course by Bela Bollobas, Foundation Books, Delhi (Cheap edition), 1994.

MAT508 – Fourier Analysis

Objectives and outcome:

Fourier Analysis is a subject which grew out of the study of Fourier series and has many scientific applications. The study of Fourier series deals with representing periodic functions by simple oscillating functions (Sines and Cosines or Complex exponentials). This representation is useful quite often in many ways. Besides, Fourier series has many applications in Electrical Engineering, Signal Processing and Quantum Mechanics.

This course is intended to provide an introduction to certain aspects of Fourier Series and related topics.

Unit I: Elementary properties of Fourier coefficients, the uniqueness theorem and the density of trigonometric polynomials, Convolution and Fourier coefficients.

Unit II: Convolution as a smoothening process, Approximate identities for convolution, Complex homomorphisms and Fourier coefficients.

Unit III: The Dirichlet and Fejer kernels, the localization principle, Uniform and mean summability and applications.

Unit IV: Convex, Quasi-convex and BV sequences, Convergence of Sine and Cosine series, Sine and Cosine series as Fourier series.

UnitV: Factorization problem, BV functions and size of its Fourier coefficients, Jordan’s test, Dini’s test, Divergence of Fourier series.

Text book:

“Fourier Series”, Vol.I - R. E. Edwards, Springer.

Sections: 2.2.4, 2.3, 2.4, 3.1, 3.2, 4.1, 5.1, 5.2, 5.3, 6.1, 6.2, 7.1, 7.2, 7.3, 7.5, 10.1, 10.2, 10.3.

<u>Note</u>: Omit the proof of 3.1.9. Communicate the main results of Ch.8 without proof.

Reference books:

(1) “Trigonometric Series”
- A. Zygmund, Cambridge University Press, 1968.

(2) “An Introduction to Harmonic Analysis”
- Y. Katznelson, Dover publications, 1968.

(3) “Fourier Analysis”
- T. W. Korner, Cambridge University Press, 1989.

MAT509EA – Mathematical Methods

Objectives and outcome:

Integral transforms are one of the usual tools in solving differential equations.The objective of this course is to introduce various integral transforms and study its applications.

Unit I: Quick review of power series method, Frobenius method, Bessel's Equation,Bessel's Functions, Bessel Functions of the second kind, Sturm-Liouville Problems, Orthogonal Functions, Orthogonal Eigenfunction Expansions.

Unit II: Laplace Transforms, Inverse Transforms, Linearity, Shifting, Transforms of Derivatives and Integrals, Differential Equations, Unit Step Function, Second Shifting Theorem, Dirac's Delta Function, Differentiation and Integration of Transforms, Convolution, Integral Equations. Solutions of differential equations using Laplace transforms.

Unit III : A Quick introduction to the theory of Fourier series, Half range Fourier series expansions, approximation by trigonometric polynomials, introduction to Fourier integrals, Fourier cosine and sine integrals, Evaluation of integrals using Fourier integral representations, Fourier cosine and sine transform, Fourier transforms and its inverse, linearity and Fourier transform of derivatives, Convolution theorem, Solution of one dimensional wave equation and heat equation by Fourier series method, Solution of one dimensional heat equation by Fourier integrals and transforms .

Unit IV: Introduction and properties of Z-transform, change of scale and shifting property. Inverse Z-transforms, Multiplication and division by K, Initial and Final value, Partial sums, Convolution property of Casual sequence, Inverse of Z-transform by division, binomial expansion and partial fractions, Inversion by Residue Method, Solution of difference equations.

Unit V: Introduction and properties of Hankel transform, Inversion formula, Parseval's theorem, Henkel's transform of the derivative of the function, Application to boundary problems, Finite Hankel transmission.

Text Books:

For Unit I, II & III:

Advanced Engineering Mathematics, Erwin Kreyszig, Wiley Student Edition, Eighth Edition 1999.

For Unit IV & V:

Advanced Engineering Mathematics, H. K. Dass, S. Chand 2006.

Reference books:

1. Mathematical methods for physicists 6th edition by Arfken & Weber (Academic press Indian Reprint).
2. Methods of Mathematical Physics (Vol. I & II) by Courant & Hilbert (1953) – New York Interscience.

MAT509EB – Problem Solving-II

Objectives and outcome:
The objective of this paper is to develop and enhance the problem solving skills of the students. The focus will be on using the theory results skillfully to solve the mathematical exercises. The students opting for this paper are expected to have good understanding of Mathematics.

Unit I: **Problems from:** Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation. Metric spaces, compactness, connectedness. Normed Linear Spaces. Spaces of Continuous functions as examples.

Unit II: **Problems from:** Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Unit III: **Problems from:** Algebra of complex numbers, the complex plane, polynomials, Power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Unit IV: **Problems from:** Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.

Unit V: **Problems from:** Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques: Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.

Reference:
Model question papers available at http://csirhrdg.res.in/csirnetmqs.htm

MAT510 - Quantitative Techniques

Objectives and Outcome:

The objective of this course is to explore various techniques to solve real life problems.

Unit 1: Probability, conditional probability, Independence of events, Mutually exclusive events, random variables and their distributions such as Binomial, Poisson, Negative binomial, Rectangular, Exponential, Normal, Beta and Gamma and Cauchy. Moments of these distributions.

Unit II: Inventory theory: Concept, classification of inventory models, EOQ model, EPQ model, EOQ model with shortages, EOQ model with constraints.

Unit III: Queueing models: Introduction, Queueing components, $((M/M/1): (\infty/FCFS))$, $((M/M/1): (N/FCFS))$, $((M/M/c) : (\infty/FCFS))$, $((M/E^k/1): (\infty/FCFS))$, $((M/M/R) : (K/GD))$, $K > R$.

Unit IV: Replacement models: Introduction, types of replacement models, replacement of items those deteriorate, replacement of items which fails completely.

Unit V: Simulation and its applications in solving queueing and inventory problems.

Text Book:

Syllabus is roughly covered by “OPERATIONS RESEARCH” by Nita H. Shah, Ravi M. Gor and Hardik Soni, PHI Publications, New Delhi, 2007.

Reference Books:

1. ANALYSIS OF INVENTORY SYSYTEMS by G. Hadley and T. M. Whitin, Prentice Hall.
2. INVENTORY SYSTEMS by E. Naddor, John Wiley Publications, 1964.
3. SIMULATION by Gordan, Prentice Hall.

MAT511EA – Differential Geometry-II

Objectives and outcome:
The objective of the course is to understand the celebrated Gauss-Bonnet theorem.

Unit I: The Second Fundamental Form, The curvature of curves on a surface, The normal and Principal Curvatures, Geometric Interpretation of Principal Curvatures.

Unit II: The Gaussian and Mean Curvatures, The Pseudosphere, Flat surfaces, Surfaces of Constant Mean Curvature, Gaussian Curvature of Compact surfaces, The Gauss map.

Unit III: Definition and Basic Properties of Geodesics, Geodesic Equations, Geodesics on Surfaces of Revolution, Geodesics as Shortest Paths, Geodesic Coordinates.

Unit IV: Gauss's Remarkable Theorem (Gauss's theorem Egregium), Isometries of Surfaces, The Codazzi-Mainardi Equations, Compact Surfaces of Constant Gaussian Curvature.

Unit V: Gauss-Bonnet theorem for simple closed curves, Gauss-Bonnet theorem for Curvilinear Polygons, Gauss-Bonnet theorem for Compact surfaces, singularities of vector fields, critical points.

Text book: "Elementary Differential Geometry", Andrew Pressley, Springer, 2001.

Reference books:

(1) 'Differential Geometry of curves and surfaces'
- Manfredo P. do Carmo. Prentice-Hall, Inc. New Jersey.

(2) 'Elements of Differential Geometry'
- R S Millman and G D Parker, Prentice Hall.Inc. New Jersey.

(3) 'Differential Geometry of Three Dimensions'
- C E Weatherburn. ELBS. Low-priced Textbook. Cambridge University Press.

(4) 'Elementary Differential Geometry' - Barrett O'Neill, Academic Press. N.Y.

MAT511EB - Insurance Models

Objectives and Outcome:

The objective of this course is to study mathematics of insurance. The preliminary of financial mathematics is introduced to analyze Cramer – Lundberg model.

Unit I: Discrete probability: Probability spaces, Independence, Binomial probabilities, Random Variables, Expectation, Variance and Standard Deviation, Covariance and Co-relation, Best Linear Predictor. Conditional Probability, Partitions and Measurability, Conditional expectation, Filterations and Martiangles.

Unit II: Portfolio Management and the Capital Asset, Pricing Model : Concept of Portfolios, Returns and Risk, Two – Asset Portfolios, Multi-Asset Portfolios, Stock Options, Profit and Pay-off curves, Selling Sort.

Unit III: Discrete time pricing Models : Assumptions, Positive Random Variables, the Basic Model, Portfolios and Trading strategies, the Pricing Problem, Arbitrage Trading Strategies, Computing Martiangle Measures.

Unit IV: Introduction and main assumptions, Poisson Model: Preliminaries, Inter-arrival and waiting time distributions.

Unit V: Order statistics property, Cramer – Lundberg Model.

Text books:

1. The Units 1 - 3 are covered from the book "INTRODUCTION TO THE MATHEMATICS OF FINANCE: FROM RISK MANAGEMENT TO OPTION PRICING" by Steven Roman, Springer Publishers.

2. The Units 4 and 5 are roughly covered from the book "Lectures in Insurance Models" by S. Ramasubramanian, 2009, Hindustan Book Agency.

Reference Books:

1. "A course in Financial Calculus" by A. Etheridge, Cambridge University Press, 2002.
2. "An elementary Introduction to Mathematical Finance", Cambridge University Press, 2002.

MAT512PT - Project

Every student is required to prepare and submit the project before the commencement of the external examinations of the fourth semester. The topic of the project for each student would be decided jointly by the student, the concerned teacher and the head of the department.