Seat No. : _______________

AE-125

April-2023

B.Sc., Sem.-VI

CC-310 : Mathematics

Time : 2½ Hours] **[Max. Marks : 70**

Instructions : (i) Attempt **all** questions.

(ii) Notations are usual everywhere.

(iii) The right hand side figures indicate marks of the sub question.

1. (a) Define the following terms and give an example for each : **7**

(i) Bipartite Graph

(ii) Induced Subgraph

(iii) Edge deleted subgraph

(iv) Underlying simple graph

(b) Define isomorphism of graphs. Show that the following graphs(Fig. – 1) are isomorphic. **7**

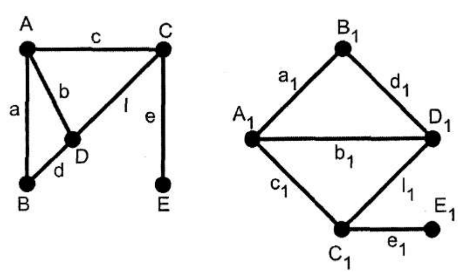


Fig. – 1

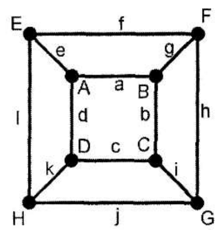
OR

(a) Given any two vertices u and v of a graph G, prove that every $u-v$ walk contains a $u-v$ path. **7**

(b) Prove that the complete graph K_n has $\frac{n(n-1)}{2}$ edges. **7**

2. (a) If T is a tree with n vertices then prove that it has precisely n – 1 edges. **7**

(b) Define the adjacency and incidence matrices of a graph and write both for the following-graphs (Fig. – 2) : **7**



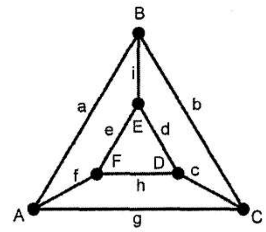


Fig. – 2

OR

(a) Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in graph G. **7**

(b) Let G be a graph with n vertices v_1, v_2,.... v_n and let A denote the adjacency matrix of G w.r.t. this listing of the vertices. Let k be any positive integer and let A^k denote the matrix multiplication of k copies of A. Then prove that $(i, j)^{th}$ entry of A^k is the number of different $v_i - v_j$ walks in G of length k. **7**

3. (a) Prove that a graph G is connected if and only if it has a spanning tree. **7**

(b) Give a list of all spanning trees, including isomorphic ones, of the complete graph K_4. **7**

OR

(a) Prove that, if a vertex v of a connected graph G is a cut vertex of G then, there are two vertices u and w of G different from v such that v is on every u – w path in G. **7**

(b) For the following graph (Fig. – 3), find (i) all cut vertices (ii) all bridges (iii) a spanning tree (draw it) (iv) connectivity and (v) all n for which it is n-connected. **7**

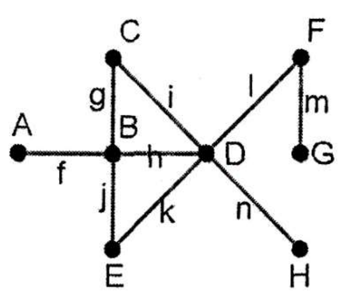


Fig. – 3

4. (a) Prove that a connected graph G is Euler if and only if the degree of every vertex is even. **7**

(b) Find closure of the graph (Fig – 4) : **7**

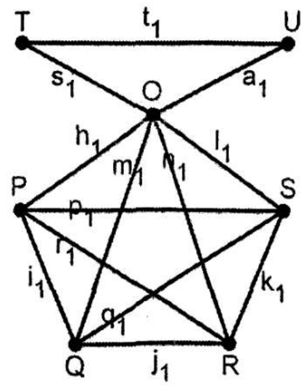


Fig. – 4

OR

(a) Write a short note on Königsberg seven bridges problem. **7**

(b) Define closure of a graph. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian. **7**

5. Attempt any **SEVEN** of the followings in short : **14**

(i) Find the graph G – {O} for the graph G of above Q.4(b) (Fig. – 4).

(ii) Define forest. Is every tree a forest ? Why ?

(iii) Draw fusion graph from the graph in (Fig. – 4) by fusing vertices T and O.

(iv) If connected graph G has 201 edges what is the maximum possible number of vertices in G ? Why ?

(v) Define a bridge and give an example.

(vi) A graph G is acyclic graph and it has 6 vertices and 5 edges. Is it tree ? Why ?

(vii) Define connectivity and n-connected graph.

(viii) A connected graph has a cut vertex. Must it be 2-connected ? Why ?

(ix) Define minimal spanning tree.

(x) Give an example of the maximal non Hamiltonian graph.

(xi) Find a Hamiltonian cycle in graph (Fig. – 4) (if exists).

(xii) Find an Euler tour in the graph (Fig. – 4) (if exists).
