

## Integ. M.Sc Sem-1 (Compu. Sci.) Examination

## Mathematical Foundations

March 2022

[Max. Marks : 70

Time : 3-00 Hours]

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**Instructions: Draw Diagrams wherever necessary.  
Make Assumptions wherever necessary.**

SECTION – I

- Q-1 Evaluate the following terms: 12
- Find the value of  $x$  and  $y$ , if  $(2x-3, y+1) = (x+5, 7)$ .
  - $A = \{a, b, c\}$  and  $B = \{1, 2, 3\}$ . Find  $(A \cup B) \times C$ .
  - Intercept of the line:  $x \cdot \sec \alpha - y \cdot \operatorname{cosec} \alpha = 5$ .
  - The slope of line to the positive direction of X-axis is  $30^\circ$ .
  - If  $f: R \rightarrow R$ ,  $f(x) = [x]$  then find  $f(2.5)$  and  $f(-2.5)$ .
  - Let  $A = \{1, 2\}$ ,  $B = \{3, 6\}$  and  $f: A \rightarrow B$  given by  $f(x) = x^2 + 2$ ,  $g: A \rightarrow B$ ,  $g(x) = 3x$ . Check whether  $f = g$  or not.

- Q-2 Attempt the following: 12
- What the bit string represents the set of all integers that do not exceed 5? And write the string of the complement of this set? ( $U = \{1, 2, 3, \dots, 10\}$ )
  - Find the equation of line passing through the points  $(-2, 4)$  and  $(1, 2)$ .
  - If  $f: R \rightarrow R$  is bijective function given by  $f(x) = x^3 + 3$ , then find  $f^{-1}(x)$ .
  - Find the value of  $\log_2(\log_3(\log_3 27^3))$ .

**OR**

- Q-2 Attempt the following : 12
- If  $A = \{x / x \text{ is multiple of } 2 \leq 10\}$  and  $B = \{x / x \text{ is factor of } 12 \leq 5\}$ . Then prove one of the De Morgan's law.
  - Find the area of triangle having sides 10 units, 12 units and 14 units.
  - Find angle between two lines  $x + 2y = 3$  and  $2x + 4y = 1$ .
  - If  $\frac{1}{2}(\log_a x + \log_a y) = \log_a \left( \frac{x-y}{3} \right)$ ; then prove that  $\frac{x}{y} + \frac{y}{x} = 11$ .

P. T. O.

- Q-3 Attempt the following: 11
- a. Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone. 40 of them had both, a two-wheeler and a credit card, 30 had both, a credit card and a mobile phone and 60 had both, a two-wheeler and mobile phone and 10 had all three. How many candidates had none of the three? (5 marks)

- b. Examine the continuity of function defined by, (6 marks)

$$f(x) = \begin{cases} -2x^2 & x \leq 0 \\ 5x+2 & 0 < x \leq 1 \\ 3x^2+4x & 1 < x \leq 2 \end{cases}$$

at the points  $x = 0, 1$ .

**OR**

- Q-3 Attempt the following: 11
- a. Prove that the triangle ABC is right angled triangle having points A(3,-1), B(6,2) and C(-2,4). (5 marks)
- b. If  $f: R \rightarrow R$ ;  $f(x) = 2x+1$  and  $g: R \rightarrow R$ ;  $f(x) = 3x-2$ . Find  $f \circ g$ ,  $g \circ f$ ,  $f \circ f$  and  $g \circ g$ . (6 marks)

### SECTION - II

- Q-4 Evaluate the following examples. 12

- a. Find the discontinuity of function  $f(x) = \frac{x^2+1}{3x^2-17x-20}$ .
- b.  $\lim_{x \rightarrow 0} \left( \frac{1+x}{1-x} \right)^{\frac{1}{x}}$ .
- c.  $\frac{d}{dx} (e^x \cdot \log x)$
- d.  $\frac{d}{dx} (7^{2x-1})$
- e.  $\int (\sec^2 x + x) dx$
- f.  $\int \frac{2x-3}{x^2-3x+78} dx$

- Q-5 Attempt the following: 12

- a.  $\lim_{x \rightarrow \infty} \frac{15x^3 - 3x^2 + 2}{20x^3 + 6x - 7}$
- b. If  $y = e^x \sin x$ , then prove that  $\frac{d^2y}{dx^2} - 2e^x \cos x = 0$
- c.  $y = \frac{2^x}{x^2}$  then find  $\frac{dy}{dx}$ .
- d.  $\int \log x dx$

OR

Q-5 Attempt the following:

12

a.  $\lim_{x \rightarrow 0} \frac{7^x - 5^x}{x}$

b. If  $y = e^{\sin 3x}$ , then find  $y_2$ .

c.  $\int x e^x dx$

d.  $\int_0^{\pi/2} \sin x dx$

Q-6 Attempt the following:

11

a.  $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^4 - 16}$

(5 marks)

b. Find the area bounded by  $y = x^2 - x - 2$  with X-axis.

(6 marks)

OR

Q-6 Attempt the following:

11

a.  $\int \frac{2}{x(x+2)(x+1)^2} dx$

(5 marks)

b. Find the maximum and minimum value of the function  $y = x^3 - 3x + 5$ .  
(6 marks)