Seat No. : _____

XR-117

April-2013

B.C.A (Sem. – II)

CC : 111 (Mathematical Foundation of Computer Science)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : Use of simple calculator is allowed.

1. (a) If $(G, *)$ is a group and if a, b, c are elements of G, then prove that : **7**

(i) $a * b = a * c \Rightarrow b = c$ (left cancellation law)

(ii) $b * a = c * a \Rightarrow b = c$ (right cancellation law)

OR

Let G be the set of all non-zero real numbers and let

$a * b = \frac{1}{2} ab$

Prove that $(G, *)$ is an Abelian group.

(b) Prove that every cyclic group is an Abelian. **7**

OR

Let $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ be a multiplicative group. Find order of every element.

2. (a) Let $A = \{a, b, c, d\}$ and **7**

$R = \{(a, b), (a, a), (b, a), (b, b), (c, c), (d, d), (d, e), (e, d), (e, e)\}$ and

$S = \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, c), (c, d), (d, e), (e, d)\}$ be the equivalence relations on A. Determine the partitions corresponding to $R \cap S$.

OR

Let $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ whose all elements are divisors of 100. Let the relation "$\leq$" be the relation "/" (divides) be a partial ordering on D_{100}. Determine the GLB and LUB of B, where $B = \{5, 10, 20, 25\}$.

(b) Draw the Hasse Diagram of (S_{64}, D). **7**

OR

Let $A = \{a, b, c, d\}$ and consider the relation $R = \{(a, b), (a, b), (a, c), (a, d), (b, b), (b, d), (c, c), (c, d), (d, d)\}$

Show that R is a partial ordering.

3. (a) Find the product of sum expansion of the Boolean function. **7**

$f(x, y, z) = (x + z) \cdot y$

OR

Define the following :

(i) Boolean Algebra

(ii) Direct Product of two lattice

(iii) Atoms

(iv) Complemented lattice

(v) Join Irreducible

(vi) Bounded Lattice

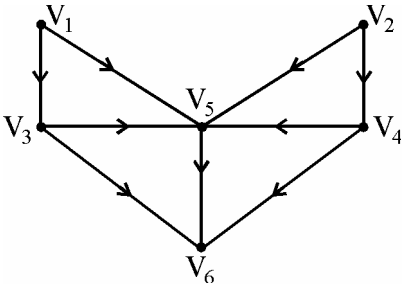
(vii) Chain

(b) Let $(L, *, \oplus)$ be a lattice. If $a, b, c \in L$ prove that $(a * b) * c = a * (b * c)$. **7**

OR

Define Sub-lattice. Find all the sub-lattice of the lattice (S_{12}, D). Is (S_{12}, D) a complemented lattice ?

4. (a) Find radius and diameter of the following graph : **7**

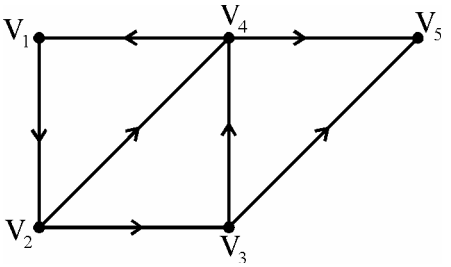


OR

Give three types of representation of the following tree :

(A(B(E((H) (I) (J))) (C) (D(F(K))(G)))

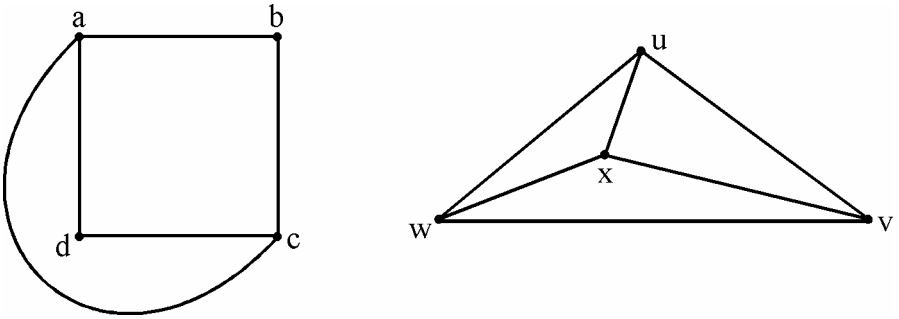
(b) Define Path, Simple Path and Elementary Path. Determine all elementary paths from V_1 to V_5 in the following graph : **7**



OR

(b) Define Isomorphic Graphs.

Determine the following graphs are isomorphic or not ?



5. Attempt any **fourteen** : **14**

(1) If '$*$' is a binary operation on any set S, then $a * a = a$. (true/false)

(2) Let S be a set having exactly 3 elements. How many different binary operations can be defined on S ?

(a) 3^2 (b) 2^3

(c) 3^{3^2} (d) 3^{2^3}

(3) Which of the following does not hold associative law for binary operations ?

(a) (N, $*$), where $x * y = 9cd\ (x, y)$ for all $x, y \in N$

(b) (N, $*$), where $x * y = 1cm\ (x, y)$ for all $x, y \in N$

(c) (R, $*$), where $x * y = |x| + |y|$ for all $x, y \in R$

(d) (N, $*$), where $x * y = x^y$ for all $x, y \in N$

(4) A group is a special type of _____.

(a) Monoid (b) Groupoid

(c) Abelian (d) None of these

(5) Let A = {1, 2, 3}. Give an example of a relation on set A which is neither symmetric nor antisymmetric.

(6) If 'xRy' stands for "x is a child of y", then state whether the relation is reflective, symmetric, antisymmetric or transitive ?

(7) Let S = {1, 2, 3, 5, 6, 10, 15} is ordered by divisibility. Out of the following which pair of elements of S are non-comparable ?

(a) (1, 2) (b) (2, 6)

(c) (2, 3) (d) None of these

(8) Let S = {1, 2, 3, 4,.....} is ordered by divisibility, then investigate which of the following subset of S is not linearly ordered ?

(a) (2, 4, 8) (b) (3, 6, 9, 11)

(c) (1) (d) None of these

(9) Find complement of the following Boolean expressions by De-Morgan's law.
& (A, B, C) = (A' + B' C)'

(10) Atoms are immediate _____ of lowest element 'O'. (predecessors/successors)

(11) Every chain is a lattice. (true/false)

(12) In a lattice if $a \leq b \Leftrightarrow a \oplus b =$ _____

(a) a (b) b

(13) A vertex of degree zero is called _____ vertex. (isolated/pendent)

(14) Draw a graph with six vertices and four edges.

(15) Define Loop in a graph theory.

(16) The graph K_7 has _____ number of edges.

(a) 21 (b) 25

(c) 28 (d) none of these

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