Seat No. : _______________

MO-128

March-2019

B.Sc., Sem.-VI

CC-309 : Mathematics

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Right hand side figure indicates marks of that question.

1. (A) (i) Let X be a metric space. Prove that A subset G of X is open if and only if it is a union of open spheres. **7**

(ii) Prove that in any metric space X, each open sphere is an open set. **7**

OR

(i) Define close set. Let X be a metric space. A subset F of X is closed if and only if its complement F' is open.

(ii) Let X be a complete metric space and let Y be a subspace of X. Prove that Y is complete if and only if it is closed.

(B) Attempt any **two** short questions : **4**

(1) Is the real function $|x|$ defined on real line R is metric ? Justify.

(2) Define metric space.

(3) Define interior of A. Give any two basic properties of Int(A).

2. (A) (i) Prove that closed subset of a compact sets are compact. **7**

(ii) Prove that a compact subset of a metric space are closed. **7**

OR

(i) A subset E of a real line R^1 is connected if and only if it has following property : "If $x \in$ E, y $\in$ E and x < z < y then $Z_0 \in$ E".

(ii) A mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

(B) Attempt any **two** short questions : **4**

(1) Define compact metric space.

(2) Define complete metric space.

(3) Define bounded mapping.

3. (A) (i) State and prove Weierstrass M-test. Show that $f_n(x) = n^2 x^n (1 - x)$; $x \in [0, 1]$ does not converges uniformly to a function which is continuous on [0,1]. **7**

(ii) Let f_n satisfy

(1) $f_n \in D\,[a, b]$

(2) $(f_n(x_0))$ converges for $x_0 \in D[a, b]$

(3) f_n converges uniformly on [a, b] then prove that f_n converges uniformly on [a, b] to a function f. **7**

OR

(i) Let (f_n) be a sequence of continuous function on $E \subset C$ converges uniformly to f on E then prove that f is continuous on E.

(ii) Prove that there exists a function $f : R \to R$ which is continuous everywhere but differentiable nowhere.

(B) Attempt any **two** short questions : **3**

(1) Is $f_n(x) = \dfrac{1}{1 + nx}$ $(x \geq 0)$ point wise convergent ? justify.

(2) If the series Σa_k converges absolutely then prove that the series $\Sigma a_k \cos(kx)$ is uniformly convergent on R.

(3) Define Uniform convergence.

4. (A) (i) Let $f(x) = \Sigma a_n x^n$ be a power series with radius of convergence 1. If the series converges at 1 then prove that $\lim\limits_{x \to 1^-} f(x) = f(1)$ **7**

(ii) State and prove Weierstrass Approximation theorem. **7**

OR

(i) For every $x \in R$, and $n > 0$, prove that

$$\sum_{k=0}^{n} (nx - k)^2 \binom{n}{k} x^k (1 - x)^{n-k} = nx(1 - x) \leq \frac{n}{4}$$

(ii) Show that for $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \ldots$ for $-1 \leq x \leq 1$. Hence deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots..$$

(B) Attempt any **two** short questions : **3**

(1) Show that $\log 2 = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots\ldots..$

(2) State Binomial series for $\alpha \in R$ and $|x| < 1$.

(3) Define Taylor's series.
