Seat No. : _______________

# ME-130

**March-2019**

**B.Sc., Sem.-V**

**304 : Mathematics**
**(Mathematical Programming)**

**Time : 2.30 Hours]** **[Max. Marks : 70**

**Instruction :** (i) **All** the **4** questions are compulsory.
(ii) Notations are usual everywhere.
(iii) The right hand side figures indicate marks of the question/sub question.

1. (a) (i) Define a convex Polyhedron and prove that a convex polyhedron is a convex set. **7**

(ii) A manufacturer produces two types of models $M_1$ and $M_2$. Each $M_1$ model requires 4 hours of grinding and 5 hours of polishing. Each $M_2$ model requires 5 hours of grinding and 3 hours of polishing.The manufacturer has two grinders and 3 polishers .Each grinder works 40 hours a week and each polisher works for 60 hours a week. Profit on $M_1$ model is ₹ 7 & on $M_2$ model is ₹ 9. Whatever is produced in a week is sold in the market.
How should the manufacturer allocate his production capacity to the two types of models so that he may make a maximum profit in a week ?
Formulate the LP problem . **7**

**OR**

(i) If $S_F$ is a non-empty set of all feasible solutions of an LP Problem then prove that $S_F$ is a convex set.

(ii) A manufacturer of furniture makes two products : chairs and tables. These products are processed on two machines A and B. A chair requires 2 hours of processing time on machine A and hours on machine B. A table requires 5 hours of processing time on machine A and no time on machine B. There are 16 hours of time available for machine A and 20 hours on machine B during a working day. Profit gained by the manufacturer from a chair is ₹ 50 and that of a table is ₹ 90. What should be the daily production of each of the two products? Formulate the linear programming problem.

(b) Attempt any **TWO** of the followings in short : **4**

(i) Define a convex set and a vertex of a convex set

(ii) Define a Convex hull of a set and find [S] if $S = \phi$.

(iii) Determine the convexity of the sets $S_1 = \phi$ and $S_2 = [0, 1]$ in Euclidean space $E_1 = R$.

2. (a) (i) Solve the following LPP by Simplex Method :

$$\text{Maximize } Z = x_1 + x_2 + 3x_3$$

$$\text{Subject to } 3x_1 + 2x_2 + x_3 \leq 3$$

$$2x_1 + x_2 + 2x_3 \leq 2 \quad \text{and} \quad x_1, x_2, x_3 \geq 0.$$ **7**

(ii) Solve the following LPP by big-M Method :

$$\text{Maximize } Z = 3x_1 + 2x_2 + 3x_3$$

$$\text{Subject to } 2x_1 + x_2 + x_3 \leq 2$$

$$3x_1 + 4x_2 + 2x_3 \geq 8 \text{ and } x_1, x_2, x_3 \geq 0.$$ **7**

**OR**

(i) Solve the following LPP by Two Phase Method :

$$\text{Maximize} \quad Z = 3x_1 + 2x_2$$

$$\text{Subject to} \quad 2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12 \qquad \text{and } x_1, x_2 \geq 0.$$

(ii) Solve the following Integer Programming Problem by the Gomory's Cutting plane Method :

$$\text{Maximize } Z = 2x_1 + 3x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 6$$

$$2x_1 + x_2 \leq 8; \quad x_1, x_2 \geq 0 \text{ and are integers.}$$

(b) Attempt any **TWO** of the followings in short : **4**

(i) Define a Basic and a basic feasible solution of a Linear programming problem.

(ii) Define a Slack Variable and an Artificial Variable

(iii) Define an Integer Programming Problem.

3. (a) (i) Explain Standard primal form of a Linear Programming Problem and describe how to find Dual of such a Linear Programming Problem with an example. **7**

(ii) Use the Dual simplex Method to solve the following LP Problem :

$$\text{Minimize } Z = 2x_1 + x_2 + x_3$$

$$\text{Subject to } 4x_1 + 6x_2 + 3x_3 \leq 8$$

$$x_1 - 9x_2 + x_3 \leq -3$$

$$-2x_1 - 3x_2 + 5x_3 \leq -4 \quad \text{and} \quad x_1, x_2, x_3 \geq 0.$$ **6**

**OR**

(i) Prove that the dual of the dual is the primal of the linear programming problem. Also verify it for the following linear programming problem :

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to $x_1 + 5x_2 - 2x_3 \leq 0$

$3x_1 + 4x_2 - 6x_3 \leq 10$

$5x_1 + 7x_2 - 8x_3 \leq 20$ and $x_1, x_2, x_3 \geq 0.$

(ii) Use the principle of Duality to solve the following LP Problem :

Minimize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to $x_1 + x_3 \geq 2$

$x_2 + x_3 \geq 5$ and $x_1, x_2, x_3 \geq 0.$

(b) Attempt any **TWO** of the followings in Short : **4**

(i) When a Dual Simplex method is applicable to solve an LP Problem ?

(ii) Describe any two advantages of Duality.

(iii) Find the Dual of following LP Problem :

Minimize $Z = 2x_1 + 4x_2$

Subject to $x_1 + x_2 \geq 4$

$x_1 - x_3 \geq 5$ and $x_1, x_2 \geq 0.$

4. (a) (i) What is an assignment problem ? **7**

Explain how is it a special case of the transportation problem.

Also describe the main differences between them.

(ii) Solve the following assignment Problem by Maximization criterion : **6**

| | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|
| **I** | 40 | 35 | 38 | 41 | 10 |
| **II** | 42 | 35 | 34 | 40 | 12 |
| **III** | 38 | 34 | 34 | 37 | 11 |
| **IV** | 12 | 14 | 11 | 10 | 9 |
| **V** | 9 | 16 | 12 | 20 | 10 |

**OR**

(i) Describe the Hungarian Method for solving an Assignment Problem.

(ii) Solve the following Transportation Problem by MODI Method :

| **ORIGINS** | **DESTINATIONS** | | | | **Supply** |
|---|---|---|---|---|---|
| | **$D_1$** | **$D_2$** | **$D_3$** | **$D_4$** | |
| **$O_1$** | 10 | 10 | 11 | 14 | 35 |
| **$O_2$** | 12 | 11 | 10 | 16 | 35 |
| **$O_3$** | 18 | 16 | 14 | 12 | 30 |
| **DEMAND** | 25 | 30 | 20 | 25 | |

(b) Attempt any **TWO** of the followings in short : **4**

(i) Find the Initial b.s.f. of the following transportation problem by North West Corner Method :

| | **$D_1$** | **$D_2$** | **$D_3$** | **$a_i$** |
|---|---|---|---|---|
| **$O_1$** | 16 | 20 | 12 | 20 |
| **$O_2$** | 14 | 8 | 18 | 20 |
| **$O_3$** | 26 | 24 | 16 | 60 |
| **$b_i$** | 30 | 40 | 30 | 100 |

(ii) Find the Initial b.s.f. of the following transportation problem by Least Cost Method :

| | **$D_1$** | **$D_2$** | **$D_3$** | **$a_i$** |
|---|---|---|---|---|
| **$O_1$** | 16 | 20 | 12 | 20 |
| **$O_2$** | 14 | 8 | 18 | 20 |
| **$O_3$** | 26 | 24 | 16 | 60 |
| **$b_i$** | 30 | 40 | 30 | 100 |

(iii) What is an unbalanced Transportation Problem ? How will you solve it ? Answer in short.