Seat No. : _______________

MB-130

March-2019

B.Sc., Sem.-V

301 : Mathematic
(Linear Algebra-II)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) Each question is compulsory.

(2) Figures to right indicate full marks to the question.

1. (A) (i) Define an annihilator. Prove that if A is a subset of real vector space V then A^0 (annihilator of A) is a subspace of V*. **7**

(ii) Find the Dual basis of the basis $B_2 = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for the vector space R^3. **7**

OR

(i) Prove that the addition of two linear maps is a linear map.

(ii) If a linear map $T : R^3 \to R^2$ is defmded as $T(x, y, z) = (x + y, 2y + z)$; $(x, y, z) \in R^3$, Then solve the operator equation $T(x, y, z) = (2, 4)$.

(B) Attempt any **TWO** (in short) : **4**

(i) Define the Dual basis.

(ii) Find N(T) for the linear transformation $T : R^4 \to R^3$ defined by $T(x, y, z, w) = (x - w, y + z, z - w)$

(iii) What is the dimension of vector space $L(R^2, R^4)$ and $L(R, R^2)$?

2. (A) (i) State and prove the Cauchy-schwarz in equality. **7**

(ii) Apply the Gram-Schmidt orthgonalization process to the basis $B_2 = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ in order to get orthogonal and orthonormal basis for vector space R^3. **7**

OR

(i) Prove that every finite dimensional inner product space has an orthonormal basis.

(ii) State the Cramer's rule and using it solve $2x + y = 2$, $3y + z = 1$ and $4z + x = 5$.

(B) Attempt any **TWO** (In short) : **4**

(i) In inner product space V, prove that $\| x \| = \| y \|$ if and only if $(x - y) \perp (x + y)$.

(ii) Find a non-zero vector orthogonal to the vector $x = (1, 2, 1)$ and $y = (4, 5, 2)$ in an inner product space R^3 with standard inner product.

(iii) State and prove triangle inequality in an inner product space V.

3. (A) (i) For matrix $A = (a_{ij}) \in M_n$, Prove that

$$\det A = \sum_{f \in S_n} (\text{sgn } f) a_{f(1)1} a_{f(2)2}, \ldots, a_{f(n)n}.$$ **7**

(ii) State (only) the Cramer's rule and using it solve $2x + y = 2$, $3y + z = 1$ and $4z + x = 5$. **6**

OR

(i) If $f = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 3\,4\,5\,1\,7\,2\,6 \end{pmatrix}$; $g = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 2\,5\,4\,6\,1\,7\,3 \end{pmatrix} \in S_7$. Then,

Find fog, $(gof)^{-1}$, sgn f and $\text{sgn}(g^{-1})$.

(ii) In usual notation, Prove that $\det(AB) = \det A \cdot \det B$.

(B) Attempt any **TWO** (in short) : **4**

(i) Define similar matrices.

(ii) If $\det A = -2$ then find $\det(A^3)$.

(iii) State laplace expansion.

4. (A) (i) Diagonalize the matrix $A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 04 & -2 & -4 \end{pmatrix}$. **7**

(ii) Prove that the eigen values of symmetric linear transformation are real. **6**

OR

(i) Prove that distinct eigen vectors of $T \in L(U, V)$ corresponding to distinct eigen values of T are linearly independent.

(ii) Verify the Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 4 & 3 \\ 4 & 6 & 2 \\ 3 & 2 & 11 \end{bmatrix}$.

(B) Attempt any **TWO** (In short) : **4**

(i) Define similar matrices.

(ii) If $\det A = -2$ then find $\det(A^3)$.

(iii) State laplace expansion.
