Seat No. : _______________

MH-103

March-2019

B.Sc., Sem.-III

201 : Statistics
(Random Variable and Probability Distribution Theory–I)
(Old Course)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Figures to the right indicate full marks of the question/sub-question.
(3) Notations used in this questions paper carry their usual meaning.

1. (A) (1) Define : Random variable. With respect to random variable, state its types, probability density function, for a random variable X. p.d.f is given as under : **7**

$$f(x) = \begin{cases} c(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find constant c, P(X < 0.5), P(0.25 < X < 0.50)

(2) Define probability distribution function of a random variable X. State and prove the properties of the distribution function. **7**

OR

(1) If a random variable X has a probability distribution as shown in the following table :

x	0	1	2	3	4	5	6
P(*x*)	C	2C	C	4C	5C	C	C

Determine (i) C, (ii) P(2), P(X > 2), P(1 < X < 4), (iii) Distribution function.

(2) Define marginal probability function and conditional probability function.

Y	X		
	1	**2**	**3**
1	0.40	0.20	0.05
2	0.05	0.10	0.20

Find marginal distribution of X = 2 and conditional distribution of Y = 1 given X = 1.

(B) Attempt any **Two** : **4**

(1) Define probability mass function and state necessary conditions for a probability function to be a probability mass function.

(2) Give one example, each of discrete and continuous types of random variables.

(3) What is independence of two random variables ?

2. (A) (1) Define mathematical expectation. In usual notations, prove that

$E(X + Y) = E(X) + E(Y)$. **7**

(2) Derive the relation between raw moments and moment generating function. **7**

OR

(1) In usual notations, prove following :

(i) $E(aX + b) = a\,E(X) + b$

(ii) $V(aX + b) - a^2\,V(X)$, where a and b be any constants.

(2) Define raw moments, central moments, moment generating function, cumulant generating function.

In usual, prove that moment generating function is independent of change of origin but not of scale.

(B) Attempt any **Two** : **4**

(1) State values of (i) $E(2x + 3)$ (ii) $E(3X^2 + 7)$, if $E(X) = 2$, $E(X^2) = 1$

(2) For two independent random variables, (X, Y), state the values of Cov (X, Y) and population correlation coefficient.

(3) State the relation between cumulants and central moments.

3. (A) (1) Stating necessary assumptions derive probability mass function of binomial distribution. **7**

(2) Derive moment generating function of Poisson distribution with parameter m. Also, find mean and variance using moment generating function. **7**

OR

(1) Derive mean and variance of hypergeometric distribution.

(2) In usual notations, show that $E(X) = np$, $V(X) = np(1 - p)$, where $X \sim Bn(n, p)$

(B) Attempt any **Three** : **3**

(1) Give one applications of hyper geometric distribution.

(2) State recurrent relation of cumulants of poisson distribution.

(3) State additive property of Poisson distribution.

(4) State mean and variance of Bernoulli distribution.

(5) Suggest the probability distribution to study dichotomous cases.

4. (A) (1) For the beta distribution of first kind, obtain its mean and harmonic mean. **7**

(2) For one parameter exponential distribution, derive moment generating function. Also, state its cumulant generating function. **7**

OR

(1) For a probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Show that $E(X) = \frac{b+a}{2}$, $V(X) = \frac{(b-a)^2}{12}$

(2) If a r.v. X has an beta distribution of second kind, obtain mean of X.

(B) Attempt any **Three** : **3**

(1) State the probability distribution function of uniform distribution.

(2) State the value of variance of beta type one distribution.

(3) State memoryless property of exponential distribution.

(4) State the probability density function of beta type two distribution.

(5) State one application of Uniform distribution.

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(Distribution Theory–I)
(New Course)

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Instructions : (1) **All** questions are compulsory.
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(3) Notations used in this questions paper carry their usual meaning.

1. (A) (1) State and prove recurrent relation for cumulants of binomial distribution. **7**

(2) State probability mass function of poisson distribution. In usual notations, show that $X + Y \sim Po(m_1 + m_2)$ where $X \sim Po(m_1)$ and $Y \sim Po(m_2)$, and are independent random variables. **7**

OR

(1) What is Truncation ? Derive truncated binomial distribution truncated at $X = 0$. Also obtain its mean.

(2) In usual notations, derive moment generating function of poisson distribution with parameter m.

(B) Attempt any **Two** : **4**

(1) State mean and variance of rectangular distribution.

(2) State recurrent relation for central moments of poisson distribution. Also, write the value of first two cumulants.

(3) Give one application, each of binomial distribution and of poisson distribution.

2. (A) (1) Identify the probability distribution a random variable X if its form is **7**

$$F(X) = \begin{cases} 0, & x \le \alpha \\ \frac{x-\alpha}{\beta-\alpha}, & \alpha < x < \beta \\ 1, & x \ge \beta \end{cases}$$

Write the probability density function and obtain mean as well as variance of a random variable X.

(2) The probability density function of a random variable X is **7**

$$f(X) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

In usual notations, derive moment generating function of a random variable X. Also find first two raw moments from moment generating function.

OR

(1) For beta type-I distribution, derive its mean and harmonic mean.

(2) For the gamma distribution with parameters (α, β), obtain, mean and mode.

(B) Attempt any **Two** : **4**

(1) State probability density function of beta type-II distribution. Also, state its mean value.

(2) If X ~ G(a, m) and Y ~ G(a, n) be two independently distributed gamma variates, then state the distributions of X/(X + Y) and X/Y.

(3) Give one application, each of uniform and exponential distributions.

3. (A) (1) Define Jacobin. State its uses in probability distribution theory. **7**

(2) If X and Y are independent random variables, in usual notations, show that the probability density function of U = X + Y is **7**

$$h(y) = \int_{-\infty}^{\infty} f(v)\, f(u - v)\, dv$$

OR

(1) Let X be a continuous random variable with probability density function f(x), If Y = g(x) is monotonically increasing or decreasing function of X, then, the probability density function of Y is $h(y) = f(x) \left| \frac{dx}{dy} \right|$.

(2) If the cumulative distribution o X is F(X), then obtain the cumulative distribution function and probability function of (i) $Y = X^2$ (ii) $Y = \sqrt{X}$

(B) Attempt any **Three** : **3**

(1) Given two random variables X and Y, state the joint probability density function of (U, V), when random variables X and Y are transformed to new random variables U and V.

(2) State the distribution of the quotient of two random variables.

(3) If the cumulative distribution o X is F(X), then, state probability function of Y = 3X + 1, where probability function of X = $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$.

(4) "It is not possible to extend the concept of transformation of two random variables to a set of n random variables." Do you agree ?

(5) If X and Y are independent random variables, in usual notations, and to derive the probability density function of U = XY, then range of U depends on which of the two variables (X, Y) ?

4. (A) (1) Define order statistics. Derive probability density function of the smallest order statistics. **7**

(2) Derive joint probability density function of order statistics. Also, state the joint probability function of the largest and the smallest order statistics. **7**

OR

(1) For an exponential distribution with the probability density function as

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

If a random sample (X_1, X_2, X_3) of size 3 is taken on X, derive probability distribution of the largest order statistics (Yn). Also find $P(Y_n < 0.3)$.

(2) Obtain compound distribution of Poisson and Gamma distributions.

(B) Attempt any **Three** : **3**

(1) State one use of order statistics.

(2) State probability density function of the smallest order statistics (Y_1).

(3) If a random sample (X_1, X_2, X_3) of size 3 is taken on $X \sim R(2, 5)$, state the possible number of order statistics.

(4) Define compound distribution.

(5) "The probability distribution of sample range is derived using only the largest order statistics". Identify the error and correct it.