Seat No. : _______________

SL-116

September-2020

B.Sc., Sem.-VI

CC-310 : MATHEMATICS
(Graph Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (i) Attempt any **THREE** questions in. Section-I.

(ii) Section-II is a compulsory section of short questions.

(iii) Notations are usual everywhere.

(iv) The right hand side figures indicate marks of the sub question.

SECTION – I

Attempt any **THREE** of the following questions :

1\. (a) Define (1) vertex deleted subgraph (2) induced subgraph induced by vertex set and find the graphs (i) G – {D, H, K} (ii) G – {c, g, j} (iii) subgraph induced by {c, g, j} for the following graph (Fig-1). **7**

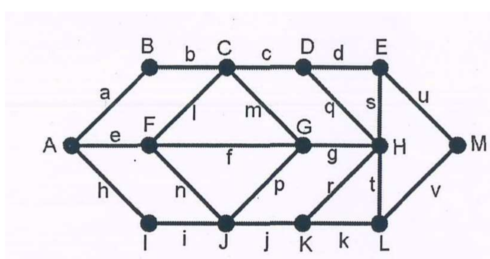

Fig. 1

(b) Define the k–cube Q_k for integer $k \geq 1$ and show that it has 2k vertices, $k2^{k-1}$ edges. **7**

2. (a) Define isomorphism of graphs. Show that the following graphs (Fig-2) are isomorphic. **7**

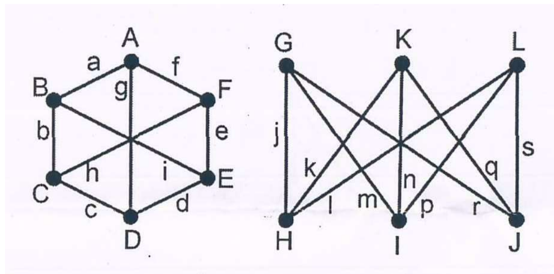

Fig. 2

(b) Prove that the complete graph K_n has $\frac{n(n-1)}{2}$ edges. **7**

3. (a) If u and v are distinct vertices of a tree T, then prove that there is precisely one path from *u* to *v*. **7**

(b) Let G be a graph with n vertices. If G is a cyclic graph with n – 1 edges, then prove that G is a tree. **7**

4. (a) Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in graph G. **7**

(b) Let e be an edge of the graph G and, as usual, let G – e be the subgraph obtained by deleting e. Then prove that $\omega(G) \leq \omega(G - e) \leq \omega(G) + 1$. **7**

5. (a) Prove that a graph G is connected if and only if it has a spanning tree. **7**

(b) Find Connectivity k(G) for the following graphs (Fig-3). If k(G) = I, identify the cut vertices. **7**

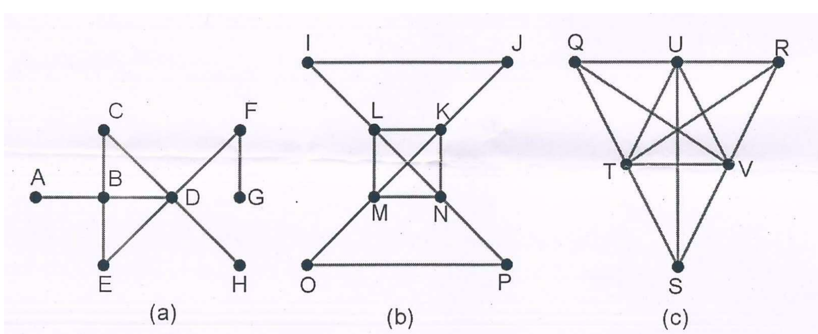

Fig. 3

6. (a) Prove that if a vertex v of a connected graph G is a cut vertex of G then, there are two vertices u and w of G different from v such that v is on every u – w path in G. **7**

(b) Let G be a graph with n vertices, where $n \geq 2$. Then prove that G has at least two vertices which are not cut vertices. **7**

7. (a) Prove that a connected graph G is Euler if and only if the degree of every vertex is even. **7**

(b) Find closure of the graph (Fig-4) : **7**

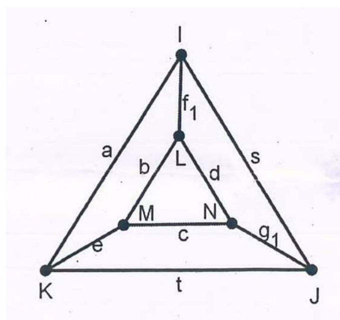

Fig. 4

8. (a) Write a short note on Königsberg seven bridges problem. **7**

(b) Use the Fleury's algorithm to produce an Euler tour for the following graph (Fig-5) **7**

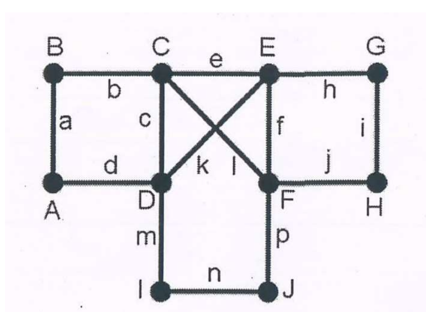

Fig. 5

SECTION – II

9. Attempt any **FOUR** of the followings in short : **8**

   (i) Define the distance between two vertices in a connected graph and find distance between A and M in (Fig-1).

   (ii) Define self-isomorphic graph and give an example.

   (iii) Draw fusion graph from the graph in (Fig-1) by fusing vertices A and F.

   (iv) Define (i) Tree and (ii) Bridge.

   (v) A graph is disconnected. What is its connectivity ? Define spanning tree.

   (vi) Define Hamiltonian Cycle. Is the graph in (Fig-5) Hamiltonian ?