Seat No. : _______________

XC-120

T.Y.B.Sc.

March-2013

Statistics Paper – IX

Time : 3 Hours] **[Max. Marks : 105**

1. (a) If X and Y are two independent Chi-square variates with parameters m and n respectively, obtain the distribution of (i) X + Y (ii) X/Y and (iii) $\frac{X}{X+Y}$.

OR

If x_i (i = 1, 2, …, n) are independent normal variates with mean zero and variance σ^2, then derive the distribution of

(i) $\sum_{i=1}^{n} x_i^2$

(ii) $\sqrt{\sum_{i=1}^{n} x_i^2}$ and

(iii) $\sum_{i=1}^{n} \frac{x_i^2}{n}$.

(b) Write in detail all applications of Chi-square distribution. Let S^2 be the variance of a random sample of size 6 from $N(\mu, 12)$, then find $P(2.3 < S^2 < 22.2)$.

OR

If $x \sim \chi_n^2$ then prove that $\lim_{n \to \infty} P\left[\sqrt{2x} - \sqrt{2n-1} \le z\right] = \Phi(z)$, where $\Phi(z)$ is the cumulative distribution function of standard normal distribution.

(c) (i) Let x_i (i = 1, 2, …, 24) be a random sample from a normal distribution mean 2 and variance 4. Compute E(S), where $S = \sum_{i=1}^{24} (x_i - \mu)^2$.

(ii) If $x_i \sim \chi_{n_i}^2$ (i = 1, …, 4) then state the distribution of $v = \frac{\sum_{i=1}^{3} x_i}{\sum_{i=1}^{4} x_i}$.

(iii) Let x and y be independent standard normal variates. What will be the distribution of $U = \left(\frac{x+y}{x-y}\right)^2$?

2. (a) Define "t" statistics and derive its probability density function. Explain its applications.

OR

Define Snedecor's F-statistic and obtain its p.d.f.

(b) Obtain the sampling distribution of the sample correlation coefficient 'r' when population correlation coefficient $\rho = 0$. Further show that when $\rho = 0$, $\frac{r}{\sqrt{1 - r^2}}\sqrt{n - 2}$ is a t-variate with $(n - 2)$ d.f.

OR

Explain how the F-distribution is related with χ^2-distribution and t-distribution.

(c) (i) Give one application of Fisher's Z transformation.

(ii) State one application of F-distribution.

(iii) The student's t-distribution with 1 degree of freedom reduces to which distribution ?

3. (a) Define Riemann-Stieltze's integral. In usual notations prove that, $f \in R(\alpha)$ on $[a, b]$ if and only if for $\forall\ \varepsilon > 0$, there exists a partition P of interval $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

OR

State and prove the theorem of "Differentiation under integral sign". If $u^3 + v^3 = x + y$ and $u^2 + v^2 = x^3 + y^3$, then show that $\frac{\partial(u, v)}{\partial(x, y)} = \frac{1}{2}\left(\frac{y^2 - x^2}{uv(u - v)}\right)$.

(b) Evaluate : (i) $\int_0^3 x^3\, dx^2$

(ii) $\int_0^3 x d[x]$, where $[x]$ is the integral part of x.

OR

Evaluate : (i) $\int_1^2 (\log x)^2\, d(\sin^{-1} \log x)$

(ii) $\int_0^1 x\, d(x^2 + 1)$

(c) (i) Define Unit-step function.

(ii) Define Polar transformation of Jacobian.

(iii) State chain rule for Jacobian.

4. (a) State and prove Dirichlet's theorem for n variables.

OR

If for h > 0, $I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-h(x^2+y^2)}dx\,dy$, then find I and hence obtain $\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)}dx$.

(b) Prove that : $\iint_D e^{-x^2-y^2}\,dxdy = \frac{\pi}{4}(1 - e^{-R^2})$ where D is the region defined by $x \geq 0$, $y \geq 0$, $x^2 + y^2 \leq R^2$.

OR

Prove that : $I = \iiiint dx\,dy\,dz\,dw$ for all values of the variables for which $x^2 + y^2 + z^2 + w^2 < b^2$ is $\frac{\pi^2}{32}(b^4 - a^4)$.

(c) (i) Give area of circle of radius r in R^2 and volume of sphere in R^3.

(ii) State the spherical polar co-ordinate transformations.

(iii) State Lioville extension of Dirichlet's Integration.

5. (a) Describe the assumptions and various steps for the construction of life table.

OR

What is abridged life table ? Explain both types of abridgement in the life table.

(b) Complete the life table of the population of a certain type of insects where x being the age in days and $l_x = 1000$ for $x = 0$.

x	0	1	2	3	4	5	6	7
q_x	0.120	0.005	0.010	0.050	0.100	0.500	0.800	0.900

OR

Complete the following life table :

Age	L_x	d_x	p_x	L_x	T_x	e_x^0
7	90000	500	?	?	4850000	?
8	?	400	?	?	?	?

(c) (i) Write only two uses of life table.

(ii) Define cohart of the life table.

(iii) Define population projection.