Seat No. : _______________

XA-131

T.Y.B.Sc.

March-2013

Statistics : Paper – VII

(Statistical Inference and Applied Statistics)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) Attempt **all** questions.

(2) **Each** question carries equal marks.

1. (a) Explain the property of unbiasedness and efficiency.

OR

Let t_n be a consistent estimator for all $\theta \in \Omega$. Let $E(t_n) = \theta_n$ and $V(t_n) \to 0$ as $n \to \infty$. Then t_n is a consistent estimator for θ.

(b) X is a uniform random variable with range $[0, \theta]$. $x_1, x_2 \dots x_n$ are independent observations on x.

Define : $\hat{\theta}_1 = \frac{2}{n}(x_1 + x_2 + \dots + x_n)$

$$\hat{\theta}_2 = \left\{\frac{(n+1)}{n}\right\} \max.(x_1, x_2 \dots x_n)$$

Show $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased for θ. Evaluate their relative efficiency.

OR

(b) Let $x_1, x_2 \dots x_n$ be a random sample from a distribution with p.d.f. $f(x, \theta) = e^{-(x - \theta)}$, $\theta < x < \infty$. Obtain sufficient statistic for θ.

(c) Answer the following objectives :

(i) Give example of a statistic t which is unbiased for a parameter θ but t^2 is not unbiased for θ^2.

(ii) Define most efficient estimator.

2. (a) State and prove Cramer-Rao inequality with regularity conditions to be stated clearly.

OR

Prove that, in general M.L.E. is consistent.

(b) Find the M.L.E. of the parameters α and λ of the distribution :

$$f(x; \alpha, \lambda) = \frac{1}{\Gamma\lambda}\left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{\frac{-\lambda x}{\alpha}}, 0 < x < \infty.$$

OR

Show that for Cauchy's distribution with parameter θ, not sample mean but sample median is consistent estimator for θ.

(c) Answer the following objectives :

(i) Give example of a MLE which is not unbiased.

(ii) State any two properties of likelihood function.

3. (a) State and prove Neymann-Pearson lemma.

OR

Describe likelihood ratio test in detail. State its properties.

(b) Obtain most powerful critical regions for testing $H_0 : \theta = \theta_0$ Vs. $H_1 : \theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$ in case of a normal population $N(\theta, \sigma^2)$ where σ^2 is known. Hence, find the power of the test.

OR

Let x_1 and x_2 be $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ respectively where the means and variance are unspecified.

Develop LR test for testing $H_0 : \mu_1 = \mu_2$ Vs. $H_1 : \mu_1 \neq \mu_2$.

(c) Answer the following objectives :

(i) Clear the difference between simple and composite hypothesis.

(ii) What is meant by most powerful test ?

4. (a) For a two-way classification state the following :

(i) Mathematical model

(ii) Null hypothesis

(iii) Assumptions

(iv) Estimation of parameters

(v) ANOVA table

OR

(a) Define analysis of variance. Give complete statistical analysis of one-way classification.

(b) Derive the formula for estimating missing observation for an $m \times m$ LSD. Give its statistical analysis.

OR

How would you derive efficiency of randomized block design over completely randomized design ?

(c) Answer the following objectives :

(i) Define : Experimental unit and experimental error.

(ii) State any two applications of completely randomized design.

5. (a) Explain fully randomized block design. State its merits and demerits.

OR

What is factorial experiment ? Explain 2^3 factorial experiment in detail.

(b) Define : Confounding. Discuss 2^3 partially confounded design.

OR

Explain fully Latin square design. State its merits and demerits.

(c) Answer the following objectives :

(i) State any two advantages of factorial experiment.

(ii) What is the importance of confounding in factorial experiment ?