Seat No. : _______________

AN-106

August-2021

B.Sc., Sem.-V

305 : Mathematics
(Discrete Mathematics)
(Elective Paper)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **3** out of first **6** questions. **7th** question is **compulsory**.
(2) Figures to the right indicate full marks of the question/sub-question.
(3) Notations used in this question paper carry their usual meaning.

1. (A) Let $\langle P, \leq \rangle$ be a poset and $A \subset P$, $A \neq \phi$. Show that $\langle A, \leq \rangle$ is a poset. **7**

   (B) Let $\langle P, R \rangle$ be a poset then prove that $\langle P, \overline{R} \rangle$ is a poset. **7**

2. (A) Show that $\langle P(X), \subseteq \rangle$ is a lattice. **7**

   (B) For a lattice $\langle L, \leq \rangle$ prove that **7**

   $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

3. (A) Define : Complete Lattice, Complemented Lattice. **7**

   Prove that every finite lattice is complete.

   (B) State and prove De' Morgan's laws in a Boolean algebra. **7**

4. (A) Prove that the direct product of any two distributive lattices is a distributive lattice. **7**

   (B) Show that in a Boolean algebra **7**

   $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$.

5. (A) Prove that the sum of all minterms in n variables is 1. **7**

(B) Find POS and SOP canonical forms of the Boolean expressions **7**

(1) $\alpha(x_1, x_2, x_3) = (x_1 * x_2) \oplus x_3$

(2) $\alpha(x_1, x_2, x_3) = x_1 \oplus x_2$

6. (A) State and prove Stone's representation theorem. **7**

(B) Let $\langle L, *, \oplus \rangle$ be a distributive lattice, for a, b, c $\in$ L, prove that **7**

$(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$.

7. Answer in short (Any **Four**) : **8**

(a) Define irreflexive relation and give an example.

(b) Define : upper bound of a set in a poset.

(c) Draw the Hasse diagram of $\langle S_{11}, D \rangle$.

(d) Find all atoms of $\langle S_6, D \rangle$ Boolean algebra.

(e) Let $\langle N, D \rangle$ be a poset. Find LUB of the subset A = {2, 3} of N.

(f) Show that $\langle S_{15}, D \rangle$ is a complemented lattice.

Seat No. : _______________

AN-106

August-2021

B.Sc., Sem.-V

305 : Mathematics
(Number Theory)
(Elective Course)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.

(2) Q-**7** is **compulsory** question of short questions.

(3) Notations are usual, everywhere.

(4) Figures to the right indicate marks of the question/sub-question.

1. (A) State and prove Division algorithm theorem. **7**

(B) Using the Euclidean algorithm to obtain the integer x and y such that

$\gcd(12378, 3054) = 12378x + 3054y$. **7**

2. (A) Find the solution of linear Diophantine equation $54x + 21y = 906$ in positive integers. **7**

(B) Prove that there are an infinite number of primes of the form $4n + 3$. **7**

3. (A) In usual notation prove that $2^{20} \equiv 1 \pmod{41}$ and find the remainder when the sum $1! + 2! + 3! + ... + 100!$ is divisible by 12. **7**

(B) Does there exists a solution of the congruence $15x \equiv 9 \pmod{12}$? If so, find out all mutually congruent solution of it. **7**

4. (A) Define congruence relations and prove that it is an equivalence relation. **7**

(B) Using Chinese remainder theorem, find integer x such that $2x \equiv 1 \pmod 3$

$3x \equiv 1 \pmod 5$; $5x \equiv 1 \pmod 7$. **7**

5. (A) State and prove Wilson's theorem. **7**

(B) If p and q are distinct primes such that $a^p \equiv a \pmod q$ and $a^q \equiv a \pmod p$, then show that $a^{pq} \equiv a \pmod{pq}$. **7**

6. (A) State and prove the Fermat's little theorem. **7**

(B) Show that $18! \equiv -1 \pmod{437}$. **7**

7. Attempt any **Four** of the followings in short : **8**

(a) If p is a prime number and p/ab then prove that p/a or p/b.

(b) A number 360 can be written as product of prime in canonical form.

(c) Define prime and relatively prime.

(d) Prove that the number N = 1571724 is divisible by 9 and 11.

(e) If $ax \equiv ay \pmod n$ and $(a, n) = 1$, then show that $x \equiv y \pmod n$.

(f) Define Euler's Phi-function.

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AN-106

August-2021

B.Sc., Sem.-V

305 : Mathematics
(Financial Mathematics)
(Elective Course)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.

(2) Q-**7** is compulsory.

(3) Notations are usual, everywhere.

(4) Figures to the right indicate marks of the question/sub-question.

1. (A) What is the Future value of ₹ 21,700 invested for ten years, for opportunity cost (interest rate) is 7% per year compounded annually, semi-annually, quarterly, monthly, weekly, daily, continuously ? **7**

   (B) Write a short Note on Time Value of Money. **7**

2. (A) What is the Future value of ₹ 1,40,000 invested for ten years, for opportunity cost (interest rate) is 5% per year compounded semi-annually, quarterly, monthly, and daily also find effective rate of interest in each case ? **7**

   (B) Derive the formulas of simple interest, and daily, weekly, monthly, quarterly, semi annually, annually, continuous compounded interest rates. **7**

3. (A) Consider a bond of n years with annual coupon payment C and face value F, if its yield (yield to maturity) is λ continuously compounded. Then derive the formula for Macaulay Duration. **7**

   (B) Consider the cash flow with annual payments of – 1000, 2000, –1000, 2000 suppose the relevant annual compound rates are finance rate is 10% and reinvestment rate 20% find MIRR. **7**

4. (A) Write a short note on comparison of NPV and IRR. **7**

(B) A company wants to immunize its bond portfolio for a targeted period of 3 years for this purpose company has decide to invest ₹ 10,00,000 at present and the details of two bonds are as follows : **7**

	Bond A	**Bond B**
Face Value	1000	1000
Market Price	986.5	1035
Macaulay Duration	5 years	2 years

Determine the amount of money invested in each bond.

5. (A) Write a short note on portfolio diagram and choice of asset. **7**

(B) Calculate the portfolios mean return and variance using the following details, **7**

$R = (0.3, 1.6, 0.9)^T$, $W = (0.3, 0.5, 0.3)$ and

$$CV = \begin{bmatrix} 1.2 & 1.4 & 0.9 \\ 1.4 & 2.2 & 0.60 \\ 0.9 & 0.60 & 1.32 \end{bmatrix} \text{ find } \bar{r} \text{ \& } \sigma^2 \text{ for portfolio.}$$

6. (A) Discuss Markowitz portfolio optimization problem with short selling and without short selling. **7**

(B) Consider a portfolio of three assets, A, B & C with the following properties. **7**

$$\overline{r_A} = 0.2,\ \overline{r_B} = 0.4,\ \overline{r_C} = 0.6,\ \sigma_A = \sigma_B = \sigma_C = 1 \ \&\ \sigma_{AB} = \sigma_{AC} = 0$$

For fixed $\bar{r} = 0.5$ find the minimum variance portfolio.

7. Attempt any **Four** of the followings in short : **8**

(a) Define inflation and write its formula.

(b) Write types of financial instrument.

(c) Define Puttable Bonds.

(d) Write the Formula for Fisher Weill Duration for discrete compounding.

(e) Define diversification in portfolio.

(f) Write the statement of two fund theorem.