3/22 **1208E179** Candidate's Seat No : _______

M.Sc. Sem-3 Examination

503 EA

Mathematics (Advance Calculus)

Time : 2-00 Hours] August 2021 [Max. Marks : 50

Instructions:

1. All the questions in **Section-I** carry equal marks.
2. Attempt any **Three** questions from **Section-I**
3. Questions in **Section-II** are **COMPULSORY**

Section-I

1. (A) Find the critical points of the function $f(x,y) = x^2 + 3y^4 + 4y^3 - 12y^2$.
   Tell whether each critical point is a local maximum, local minimum, or saddle point. **7**

   (B) Show that the equation $x^2 + 2xy + 3y^2 = 6$
   can be solved for y as a C^1 function of x near the point $(1,1)$. **7**

2. (A) Find the extreme values of $f(x,y) = 2x^2 + y^2 + 2x$
   on the set $\{(x,y) : x^2 + y^2 \leq 1\}$. **7**

   (B) Find the volume of the region above the square in the xy - plane with vertices $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ and below the surface $z = 2 - x - y$. **7**

3. (A) Find the volume of the ellipsoid $x^2 + \frac{y^2}{2^2} + \frac{z^2}{3^2} \leq 1$. **7**

   (B) Find the arc length of the curve $\mathbf{g}(t) = (2\cos t, 2\sin t, t)$, $t \in [0, 2\pi]$. **7**

4. (A) Let $(u,v,w) = \mathbf{f}(x,y) = (x+6y,\ 3xy\ ,\ x^2 - 3y^2)$.
   Compute $D\mathbf{f}(x,y)$, $\frac{\partial(u,v)}{\partial(x,y)}$, $\frac{\partial(v,w)}{\partial(x,y)}$. **7**

   (B) Evaluate $\int_{\partial S} [3x^2 \sin(y^2) dx + 2x^3 y \cos(y^2) dy]$ where S is any regular region with piece-wise smooth boundary. **7**

[P.T.O]

5. (A) Is the set $S = \{(x,y) : x^2 + 3y^2 = 3\}$ a smooth curve? 7

(B) Evaluate the iterated integral $\int_1^3 \int_1^y ye^{2x} dxdy$. 7

6. (A) Find an equation for the tangent plane to the surface

$x = e^{u-v}$, $y = u - 3v$, $z = \frac{1}{2}(u^2 + v^2)$ at the point $(1, -2, 1)$. 7

(B) Find the area of the part of the surface $z = x^2 + y^2$

inside the cylinder $x^2 + y^2 = 4$. 7

7. (A) Find the 3rd-order Taylor polynomial of $f(x,y) = x^2y + z$

based at $\mathbf{a} = (1, 2, 1)$. 7

(B) Find the centroid of the portion of the ball $x^2 + y^2 + z^2 \leq 1$

lying in the first octant $(x, y, z \geq 0)$. 7

8. (A) Let $(u, v) = \mathbf{f}(x, y) = (e^x \cos y, e^x \sin y)$.

Compute the Jacobian det $D\mathbf{f}$.

Find formulas for the local inverses of $\mathbf{f}$ when they exist. 7

(B) Compute the curl and divergence of the vector field

$\mathbf{F}(x, y, z) = xy^2\mathbf{i} + xy\mathbf{j} + xy\mathbf{k}$. 7

Section -II

1. The directional derivative of $f(x,y) = x^2y$ at the point $(2,1)$ in the direction $\left(\frac{3}{5}, \frac{4}{5}\right)$ *is* ____

(A) $\frac{7}{5}$ (B) $\frac{10}{5}$ (C) $\frac{28}{5}$ (D) 0

2. Let $g(x,y) = x^2y + 3xy + 2y^3$

$\partial_x g(1, -1) =$ ____

(A) -6 (B) -5 (C) 0 (D) 5

179-3

3. Let $\mathbf{f}(x,y) = (x-2y \;,\; 2x-y)$. The Jacobian det $D\mathbf{f} =$ ____

(A) 5 (B) 4 (C) 3 (D) 2

4. Let $u = \sin(x)\cosh(y)$, $v = \cos(x)\sinh(y)$

The images of the lines $x =$ constant in the $uv-$ plane are

(A) ellipses (B) hyperbolas (C) lines (D) parabolas.

5. The volume of the region

$S = \{(x,y,z) : |x| \leq 1, |y| \leq 1, |z| \leq 1\}$ is ____

(A) 1 (B) 2 (C) 4 (D) 8

6. A point of $\mathbb{R}^3$ has cylindrical coordinates $(1, \frac{\pi}{4}, 1)$.

Its cartesian coordinates (x,y,z) are ____

(A) $(1,1,1)$

(B) $(\sqrt{2}, \sqrt{2}, 1)$

(C) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

(D) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right)$

7. Let S be a regular region with boundary ∂S. Then the area of $S =$____

(A) $\int_{\partial S} x dy$

(B) $\int_{\partial S} y dy$

(C) $\int_{\partial S} \frac{1}{2}(xdy + ydx)$

(D) 0

8. Let $\mathbf{F}(x,y,z) = e^x \cos(y)\mathbf{i} + e^x \sin(y)\mathbf{j} + \cosh(z)\mathbf{k}$.

div$(curl\ \mathbf{F})=$ ____

(A) e^{2x}

(B) $e^x \cosh(z)$

(C) 0

(D) $e^{2x} + y\cosh(z)$

****