Seat No. : _____

JJ-110

January-2021

B.Sc., Sem.-V

CC-304 : Mathematics

(Mathematical Programming)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions from **Q-l** to **Q-8**.
(2) **Q-9** is **compulsory** question of short questions.
(3) Notations are usual, everywhere.
(4) Figures to right indicate marks of the question/sub question.

1. (A) Prove that the intersection of two convex set is a convex set. Is union of two convex sets always convex ? Justify. **7**

(B) A firm manufactures headache pills in two sizes A and B. Size A contains 2 grain of aspirin, 5 grain of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grain of bicarbonate and 6 grain of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grain of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP. **7**

2. (A) Define convex set. Consider $S_1 = \{X / | X | > l\}$. Is S_1 convex ? Justify your answer. **7**

(B) A company produces three products A,B and C. These products require three ores O_1, O_2 and O_3. The maximum quantities of the ores O_1, O_2 and O_3 available are 22 tones, 14 tones and 14 tones respectively. For one tone of each of these products, the ore requirements are :

	A	**B**	**C**
O_1	3	–	3
O_2	1	2	3
O_3	3	2	3
Profit per tone (₹ in thousand)	1	4	5

The company makes a profit of ₹ 1,000, 4,000 and 5,000 on each tone of the products A, B and C respectively. Formulate this problem as a linear programming model. **7**

3. (A) Explain Simplex algorithm for solving linear programming problem. **7**

(B) Solve the following LPP by simplex method.

Max. $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \leq 430$; $3x_1 + 2x_3 \leq 460$; $x_1 + 4x_2 \leq 420$ and $x_1, x_2, x_3 \geq 0$ **7**

4. (A) If the set of all feasible solution S_F is a non-empty bounded subset of R^n for a given Linear programming problem, Prove that the optimum solution exists at one of the vertex of S_F. **7**

(B) Solve the following LPP by two phase simplex method :

Max. $Z = 2x_1 + 3x_2 + 5x_3$ Subject to constraints

$3x_1 + 10x_2 + 5x_3 \leq 15$; $33x_1 + -10x_2 + 9x_3 \leq 33$; $x_1 + 2x_2 + x_3 \geq 4$ and $x_1, x_2, x_3 \geq 0$ **7**

5. (A) Explain concept of duality (i.e. primal-dual pair). Prove that dual of the dual is primal. **7**

(B) Apply the principle of duality to solve the following LPP. **7**

Min $Z = 2x_1 + 2x_2$

Subject to the constraint

$2x_1 + 4x_2 \geq 1$; $x_1 + 2x_2 \geq 1$; $2x_1 + x_2 \geq 1$; $x_1, x_2 \geq 0$

6. (A) State and Prove fundamental theorem of duality. **7**

(B) Using dual simplex method to solve the following Linear Programming Problem.

Max $Z = -3x_1 - 2x_2$

$x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 7$; $x_1 + 2x_2 \geq 10$; $x_2 \leq 3$; $x_1, x_2 \geq 0$ **7**

7. (A) Prove that transportation problem has a triangular basis. **7**

(B) Obtain optimum solution of the following TP. **7**

DESTINATION CENTRE

Supply centers		**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
	S_1	19	30	50	10	7
	S_2	70	30	40	60	9
	S_3	40	8	70	20	18
	D	5	8	7	14	**34**

8. (A) Explain "Hungarian method" for solving balanced Assignment Problem. **7**

(B) A department has five employees with five jobs to be performed. The time (in hours) each man will take to perform each job is given in the effectiveness matrix. **7**

	Employees					
Jobs		**I**	**II**	**III**	**IV**	**V**
	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours ?

9. Attempt any **FOUR** of the followings in Short : **8**

(a) Define a slack variable and a surplus variable.

(b) Give two examples of non-convex sets.

(c) Define a vertex. Also give another name of a vertex.

(d) When a Linear Programming Problem is said to have an Unbounded solution ?

(e) How do we solve any unbalanced assignment problem ?

(f) Give any two differences between a simple method and a dual simple method.

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