Seat No. : _______________

JH-112

January-2021

B.Sc., Sem.-V

CC-302 : Mathematics

(Analysis – I)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from questions **1** to **8**.
(2) Question **9** is compulsory question.
(3) Notations are usual everywhere.
(4) The figure to the right indicate marks of the question/sub-question.

1. (A) Prove that the set Q of all rational number is denumerable. **7**
(B) Let S be any non-empty, bounded subset of R and a be any real number then show that Sup(a + S)= a + SupS. **7**

2. (A) Prove that there does not exist a rational number r such that $r^2 = 11$. **7**
(B) If A is any set, then prove that there is no surjection of A onto the set P(A) of all subsets of A. **7**

3. (A) Prove that the sequence $\left(\left(1+\frac{1}{n}\right)^n\right)$ converges. **7**
(B) State and prove Bolzano-Weierstrass Theorem. **7**

4. (A) State and prove Sandwich theorem, using this prove that for $p \geq 2$ $\lim\limits_{n \to \infty} \frac{1}{n^p} = 0$. **7**
(B) If $s_1 = \sqrt{2}$ and $S_{n+1} = \sqrt{2s_n}$ for $n \geq 1$, prove that (s_n) is a monotonic increasing sequence bounded above and $\lim\limits_{n \to \infty} s_n = 2$. **7**

5. (A) Suppose that the function *f* is continuous on the interval [a, b]. f(a) ≠ f(b), and k is any number between f(a) and f(b). Then prove that there exists at least one point $c \in (a, b)$ such that f(c) = k. **7**
(B) Suppose g is continuous at c and f is continuous at g(c). Then prove that fog is continuous at c. **7**

6. (A) Prove that any polynomial of odd degree have at least one real root. **7**

(B) Suppose that the function f is continuous on the interval [a, b] then prove that f is uniformly continuous on [a, b]. **7**

7. (A) State and prove L'Hospital's First Rule. **7**

(B) State and prove Mean Value Theorem and verify it for $f(x) = x + |x - 1|$ on [0,3]. **7**

8. (A) State and prove Darboux's Theorem. **7**

(B) Evaluate :

(1) $\lim\limits_{x \to 0^+} \dfrac{\tan x - x}{x^3}, x \in (0, \pi)$

(2) $\lim\limits_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^x$ **7**

9. Answer any **four** of the followings in short : **8**

(A) Find the lub A and glb A of the set $A = \left\{\cos \dfrac{n\pi}{3} / n \in N\right\}$.

(B) Find the cluster points of the sequence $\{x_n\} = \{n^3\}$

(C) Find the $\lim\limits_{x \to 7} [x / 2]$ if exists.

(D) Find the derivative of $f(x) = |x^2 - 1|$.

(E) Give an example of sequence which is bounded and oscillatory.

(F) Give example of function which is nowhere continuous.