Seat No. : $\qquad$
AF-124
August-2021
B.Sc., Sem.-VI

## 311 : (Mathematics)

(Convex Analysis and Probability Theory)
Time : 2 Hours]
[Max. Marks : 50

Instructions : (i) Attempt any THREE questions in Section-I.
(ii) Section-II is a compulsory section of short questions.
(iii) Notations are usual everywhere.
(iv) The right hand side figures indicate marks of the sub question.

## SECTION-I

Attempt any THREE of the following questions :

1. (a) Define Convex and concave functions on an interval I.

Also show that the function $f: \mathrm{R} \rightarrow \mathrm{R}$ defined as $f(x)=f(x)=|\mathrm{x}|$ is decreasing on $(-\infty, 0]$, increasing on $[0, \infty)$ and convex on $(-\infty, \infty)$.
(b) Show that the function $f: \mathrm{R} \rightarrow \mathrm{R}$ defined as $f(x)=x^{2}$ is monotonically increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$.
2. (a) State and prove the Intermediate Value Theorem.
(b) Let I be an interval containing more than one point and $f: \mathrm{I} \rightarrow \mathrm{R}$ be differentiable then prove that $f^{\prime}$ is nonnegative throughout $\mathrm{I} \Leftrightarrow f$ is monotonically increasing on I.
3. (a) Define terms : Sample space, impossible and Certain Events, mutually exclusive and exhaustive events, difference events. Also, state the probability of certain events and complementary events.
(b) A balanced die is tossed twice. Write the elements of the following :
(i) Sample space.
(ii) $\mathrm{A}=$ Event that sum of the integers on two dice is 7 or 10 .
(iii) $\mathrm{B}=$ Event that integers on dice are odd.
(iv) $\mathrm{C}=$ Event that sum of integers on two dice is divisible by 3 .
(v) $\mathrm{D}=$ Event that sum of integers on two dice is greater than 6 .

Check whether events A and D are mutually exclusive or not. Also, find probabilities of events A, B, C and D.
4. (a) Define Classical definition of probability. State additive rule of probability for two and three events. If two events $\mathrm{A}, \mathrm{B}$ and C are mutually exclusive events, then state the values of $\mathrm{P}[\mathrm{AUB}]$ and $\mathrm{P}[\mathrm{AUBUC}]$. Given two events A and B such that $\mathrm{P}[\mathrm{A}]=0.32, \mathrm{P}[\mathrm{B}]=0.50$ and $\mathrm{P}[\mathrm{AUB}]=0.75$, then find $\mathrm{P}[\mathrm{A} \cap \mathrm{B}]$.
(b) Two balanced dice were tossed once. Write sample space and find the probability of the following events :
(i) 3 or more on a first die and 6 on a second die
(ii) 1 on first die and a multiple of 2 on second die.
(iii) Sum of numbers on two dice is 9
(iv) Sum of numbers on two dice is divisible by 7 .
5. (a) Stating conditions for deriving binomial distribution, state its probability function of binomial distribution. If the mean and variance of binomial distribution are 18 and $1 / 3$, then, find parameters of binomial distribution.
(b) If a random variable X follows Poisson distribution with parameter $\mathrm{m}=2$, state values of mean $(E(X))$, variance $(V(X)), E(X+4), E(2 X-3), V(4 X)$. Also, find $\mathrm{P}(\mathrm{X}=1), \mathrm{P}(\mathrm{X}<2)$.
6. (a) If a random variable $X$ follows binomial distribution with parameters ( $n, p$ ) then, state the conditions to get Poisson distribution from binomial distribution. State the mean and variance of Poisson distribution.
If, a random variable $X$ follows Poisson distribution with parameter $\theta$, then, find
$\theta$ such that $\mathrm{P}[\mathrm{X}=2]=\mathrm{P}[\mathrm{X}=3]$. Also, if $\theta=2$, find $\mathrm{P}[\mathrm{X}=0], \mathrm{P}[\mathrm{X}=1]$
$\left\{\mathrm{e}^{-1}=0.368, \mathrm{e}^{-2}=0.135, \mathrm{e}^{-3}=0.050\right\}$
(b) State the probability function of a normal distribution.

Also, state the relationship between mean, median and mode of a normal distribution. Do you agree that the normal distribution is symmetric one?

## SECTION-II

7. Answer ANY FOUR of the followings in Short :
(i) Give examples each one of convex and non-convex sets of $\mathrm{R}^{2}$.
(ii) If $\mathrm{A}=\left\{(\mathrm{x}, \mathrm{y}) \in \mathrm{R}^{2} / x^{2}+\mathrm{y}^{2} \leq 5\right\}$ then find the convex hull of A .
(iii) Define affine and convex sets.
(iv) State Bayes' theorem on probability. Also, state its uses.
(v) Give one application, each of binomial and Poisson distributions.
(vi) State moment generating function of binomial distribution.

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## AF-124

August-2021
B.Sc., Sem.-VI

311 : Mathematics
(Operations Research)

Time : 2 Hours]
[Max. Marks : 50

Instructions : (i) Attempt any THREE questions in Section-I.
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(iii) Notations are usual everywhere.
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## SECTION-I

1. (A) Discuss Economic Order Quantity (EOQ) Model with Constant rate of demand.
(B) Discuss Economic Order Quantity (EOQ) Model with Shortages.
2. (A) Discuss Economic Order Quantity (EOQ) Model with finite replenishment rate.
(B) A company plans to consume 760 pieces of a particular component. Pat records Indicates that the purchasing department spent ₹ 12,555 for placing 15,500 Purchase orders. The average inventory was valued at ₹ 45,000 and the total Storage cost was ₹ 7650 which included wages, taxes, rent, insurance etc. related to the store department. The company borrows capital at the rate of $10 \%$ per year. If the price of component is ₹ 12 and the lot-size is 10 , find the following : (1) Purchase price per year (2) Purchase expenses per year (3) Storage expenses per year (4) Capital cost per year (5) Total cost per year.
3. (A) Compare and contrast CPM and PERT. Under what conditions would you recommend scheduling by PERT? Justify your answer with reasons.
(B) Draw an arrow diagram showing the following relationships.

| Activity | A | B | C | D | E | F | G | H | I | J | K | L | M | N |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Immediate <br> Predecessor | - | - | - | A,B | B,C | A,B | C | D,E,F | D | G | G | H,J | K | I,L |

4. (A) Explain the basic logic of arrow networks.
(B) An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors selected on a geographical basis. Market research has indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned :

| Activity | Description | Time (Weeks) |
| :---: | :--- | :---: |
| A | Organize sales office | 6 |
| B | Hire salesmen | 4 |
| C | Train salesmen | 7 |
| D | Select advertising agency | 2 |
| E | Plan advertising campaign | 4 |
| F | Conduct advertising campaign | 10 |
| G | Design package | 2 |
| H | Setup packaging facilities | 10 |
| I | Package initial stocks | 6 |
| J | Order stock from manufacturer | 13 |
| K | Select distributors | 9 |
| L | Sell to distributors | 3 |
| M | Ship stocks | 5 |

5. (A) Explain Minimax and Maximin principle used in the game theory.
(B) Let the payoff matrix is as follow : $\left[\begin{array}{cc}4 & -8 \\ 5 & -2 \\ 2 & 5\end{array}\right]$. Determine optimal strategies and value of the game.
6. (A) Explain Dominance Principle in game theory.
(B) Solve the following game whose payoff matrix is given by :

$$
\left[\begin{array}{ccccc}
3 & -1 & 4 & 6 & 7 \\
-1 & 8 & 2 & 4 & 12 \\
16 & 8 & 6 & 14 & 12 \\
1 & 11 & -4 & 2 & 1
\end{array}\right]
$$

7. Attempt any FOUR short questions :
(1) Give types of direct inventory.
(2) Explain any two cost involved in inventory problem.
(3) Explain Looping and Dangling.
(4) Explain Dummy activity.
(5) Give list applications of PERT and CPM techniques in Project Management.
(6) Define two person zero sum game.
