4/22

2710E502

Candidate's Seat No :_______

B.Sc. Sem-6 Examination

310

Mathematics

October 2021

Time : 2-00 Hours] **[Max. Marks : 50**

Instructions:

(i) Attempt any THREE questions in Section-I.
(ii) Section-II is a compulsory section of short questions.
(iii) Notations are usual everywhere.
(iv) The right hand side figures indicate marks of the sub question.

SECTION-I

Attempt any THREE of the following questions :

Q.1 **(a)** State and prove first theorem of Graph Theory. Prove that in any graph G there is an even number of odd vertices. [7]

(b) Let G be the following graph(Fig-1). [7]

i. Find a closed walk of length 6. Is your walk a trail?
ii. Find an open walk of length 12. Is your walk a path?
iii. Find a closed trail of length 6. Is your trail a cycle?

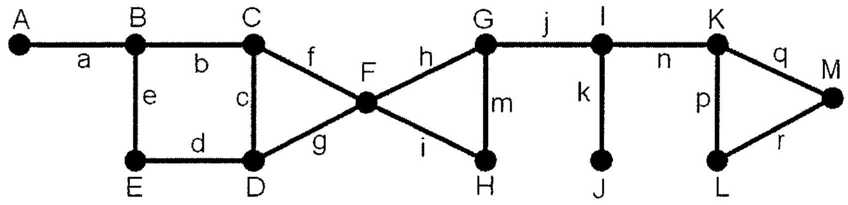

Fig- 1

Q.2 **(a)** Prove that the complete graph K_n has $\frac{n(n-1)}{2}$ edges. [7]

(b) Define isomorphism of graphs. Show that the following graphs(Fig-2) are isomorphic. [7]

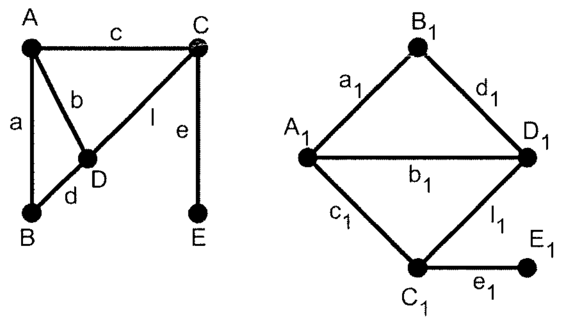

Fig- 2

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E502-1

Q.3 (a) Write down the adjacency and incidence matrices of the following graph (Fig-3). [7]

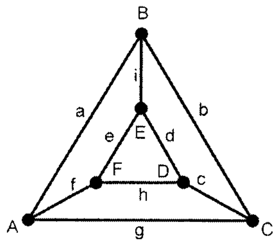

Fig- 3

(b) Let G be a graph with n vertices $v_1, v_2, \ldots, v_n$ and let A denote the adjacency matrix of G w.r.t. this listing of the vertices. Let $B = [b_{ij}]$ be the matrix $B = A + A^2 + \cdots + A^{n-1}$. Then prove that G is connected iff B has no zero entries off the main diagonal. [7]

Q.4 (a) Without drawing the actual graph, determine whether the graph G is connected or not, whose adjacency matrix is $A(G) = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$. [7]

(b) If T is a tree with n vertices then prove that it has precisely $n - 1$ edges. [7]

Q.5 (a) Let G be a simple graph with at least three vertices. Prove that G is 2-connected iff for each pair of distinct vertices u and v of G, there are two internally disjoint $u - v$ paths in G. [7]

(b) Give a list of all spanning trees, including isomorphic ones, of the connected graph (Fig-4): [7]

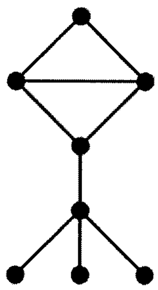

Fig- 4

Q.6 (a) Prove that a graph G is connected if and only if it has a spanning tree. [7]

(b) Find Connectivity $k(G)$ for the following graphs (Fig-5 (a), (b) and (c)). If $k(G) = 1$ identify the cut vertices. [7]

E 502-3

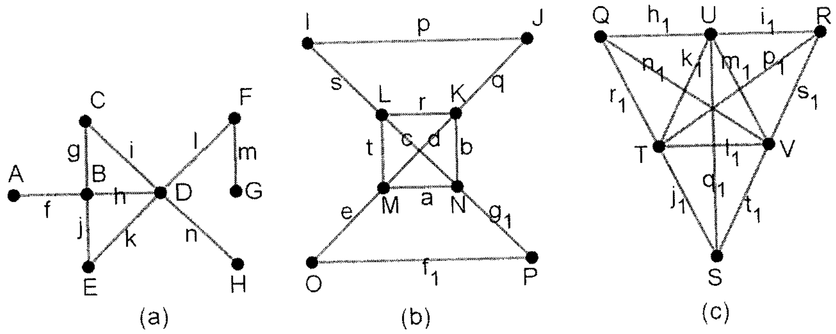

Fig- 5

Q.7 **(a)** If G is a graph in which the degree of every vertex is at least two then prove that G contains a cycle. [7]

(b) Use the Fleury's algorithm to produce an Euler tour for the following graph (Fig-6)

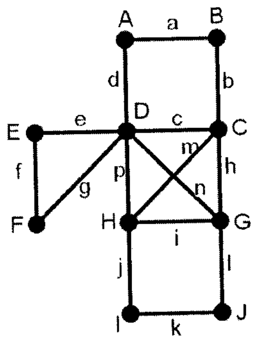

Fig- 6

Q.8 **(a)** If G is simple graph with n vertices, where $n \geq 3$, and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G, then prove that G is Hamiltonian. [7]

(b) Find closure of the graph (Fig-7): [7]

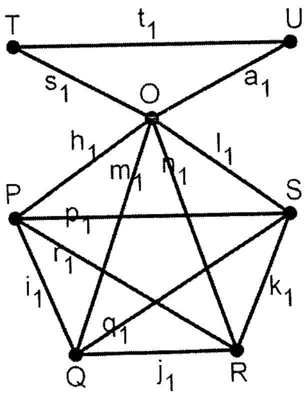

Fig- 7

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E 502-4

SECTION-II

Q.9 Attempt any FOUR of the followings in short : [8]

(i) Define regular graph and give an example.

(ii) Define a trail and path.

(iii) Draw fusion graph from the graph in (Fig-6) by fusing vertices A and C.

(iv) Define (i) Forest and (ii) Bridge.

(v) Define minimal spanning tree.

(vi) Define Hamiltonian graph and give an example.

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