Seat No. : _______________

NF-123

November-2021

B.Sc., Sem.-V

EC-305 : Mathematics

(Discrete Mathematics)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.

(2) Q-**7** is compulsory.

(3) Notations are usual, everywhere.

(4) Figures to the right indicate marks of the question/Sub-question.

1. (A) State and prove Modular Inequality. **7**

   (B) Explain Hass Diagram and also draw the Hass Diagram of (S_{105}, D). **7**

2. (A) Let X = ℕ, and relation D is defined on ℕ as, "xDy means x divides y" $\forall\ x, y \in \mathbb{N}$. Then show that $\langle \mathbb{N}, D\rangle$ is Partially Ordered Set (POSET) but not a chain. **7**

   (B) Prove (1) $a * (a \oplus b) = a$ (2) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$. **7**

3. (A) State De' Morgan's laws and prove any one of them. **7**

   (B) Define direct product of lattice and draw the Hass diagram of $\langle S_9 \times S_4, D\rangle$s **7**

4. (A) Prove that every chain is Distributive Lattice. **7**

   (B) For a complemented distributive lattice $\langle L,*,\oplus, 0, 1\rangle$s and for every $a, b \in L$, $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b' = 1$. **7**

5. (A) Let $\langle B,*,\oplus, ', 0, 1\rangle$ be a Boolean algebra. Then a non-empty element a of B is an atom of B if and only if either $a * x = 0$ or $a \oplus x = a$; $\forall x \in B$. **7**

   (B) Express $x_1 \oplus x_2$ as sum of product (SOP) canonical form in three variables. **7**

6. (A) Let $\langle B, *, \oplus, ', 0, 1\rangle$ be a Boolean algebra with n variables $x_1, x_2, ... x_n$ then **7**

(1) There are 2^n minterms in the n variables namely minj;j = 0, 1, 2, ... $2^n - 1$.

(2) $m_i * m_j = 0; \forall i \neq j$ & i, j = 0, 1, 2, ... $2^n - 1$.

(3) $\overset{2n-1}{\underset{i=0}{\oplus}} m_i = 1.$

(B) In any Boolean algebra show that $a = 0 \Leftrightarrow ab' + a'b = b$. **7**

7. Attempt any **four** of the following in short : **8**

(1) For a Lattice $(L, \leq)$, prove that $a \leq b \Leftrightarrow a * b = a$.

(2) Give a relation on the set which is Irreflexive and Transitive but not Symmetric.

(3) Find complement of each element in the set of divisors of 18.

(4) Define : Lattice homomorphism.

(5) State : Stone representation theorem.

(6) Define : Equivalent Boolean Expression.

Seat No. : _______________

NF-123

November-2021

B.Sc., Sem.-V

EC-305 : Mathematics

(Number Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) **All** Questions in Section – I carry equal marks.
(2) Attempt any **three** questions in Section – I.
(3) Question – **7** in Section – II is Compulsory.

Section – I

1. (A) State and prove Division algorithm theorem. **7**

(B) Find the all positive solutions in the integers for the Diophantine equation $24x + 138y = 18$. **7**

2. (A) Prove that the linear Diophantine equation $ax + by = c$ has a solution iff $d \mid c$, where $d = \text{g.c.d.}(a, b)$. Also prove that if x_0, y_0 is a solution of this equation then all other solutions are given by $x = x_0 + \left(\frac{b}{d}\right)t$; $y = y_0 - \left(\frac{a}{d}\right)t$
where t is any integer. **7**

(B) Using the Euclidean algorithm to obtain the integer x and y such that gcd(l2378, 3054) = $12378x + 3054y$. **7**

3. (A) Define "Congruence modulo relation for a fixed positive integer n". Also prove that it is an equivalence relation. **7**

(B) Using the Sieve of Eratosthenes find all primes $p \leq 120$. **7**

4. (A) Let $n > 0$ be fixed and a, b, c are integers then prove that if $a \equiv b \pmod{n}$, $c \equiv d \pmod{n} \Rightarrow ac \equiv bd \pmod{n}$ and $a^k = b^k \pmod{n}$ for any positive integer k. **7**

(B) Using Chinese remainder theorem, find integer x such that $2x \equiv 1 \pmod{3}$ $3x \equiv 1 \pmod{5}$; $5x \equiv 1 \pmod{7}$. **7**

5. (A) State and prove Wilson's theorem. **7**

(B) Solve the linear congruence $25x \equiv 15 \pmod{29}$. **7**

6. (A) State and prove the Fermat's little theorem. **7**

(B) (i) Find the remainder when the sum $1! + 2! + 3! + ... + 100!$ is divisible by 12.

(ii) Find the remainder when $7^{234} + 4^{111}$ is divisible by 5. **7**

Section – II

7. Attempt any **FOUR** : **8**

(1) If p is a prime number and p/ab then prove that p/a or p/b.

(2) A number 360 can be written as product of prime in canonical form.

(3) Prove that the number N = 1571724 is divisible by 9 and 11.

(4) If $ax \equiv ay \pmod n$ and $(a, n) = 1$, then show that $x \equiv y \pmod n$.

(5) Define Euler's Phi-function.

(6) State (Only) Euler's theorem.

Seat No. : _______________

NF-123

November-2021

B.Sc., Sem.-V

EC-305 : Mathematics

(Financial Mathematics)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.

(2) Q-**7** is compulsory.

(3) Notations are usual, everywhere.

(4) Figures to the right indicate marks of the question/sub-question.

1. (a) Write a short Note on Time Value of Money. **7**

(b) What is the Future value of ₹ 21,000 invested for 10 years, for opportunity cost (interest rate) is 8% per year compounded annually, semi-annually, quarterly, monthly, weekly, and continuously ? **7**

2. (a) Define shares, bonds, index and arbitrage also write no arbitrage principle. **7**

(b) What is the Future value of ₹ 40,000 invested for 7 years, for opportunity cost (interest rate) is 5% per year compounded semi-annually, quarterly, monthly, and daily ? Also find effective rate of interest in each case. **7**

3. (a) Write a short note on comparison of NPV and IRR. **7**

(b) Consider the cash flow with annual payments of 1000, –2000, –1000, 2000. Suppose the relevant annual compound rates and finance rate is 10% and reinvestment rate 15%. Find MIRR. **7**

4. (a) Consider a bond of n years with annual coupon payment C and face value F, if its yield (yield to maturity) is λ continuously compounded. Then derive the formula for Macaulay Duration. **7**

(b) A company wants to immunize its bond portfolio for a targeted period of 3 years for this purpose company has decided to invest ₹ 1,00,000 at present and the details of two bonds are as follows. **7**

	Bond A	**Bond B**
Face Value	1000	1000
Market Price	986.5	1035
Macaulay Duration	4 years	2 years

Determine the amount of money invested in each bond.

5. (a) Discuss Markowitz portfolio optimization problem with short selling and without short selling. **7**

(b) Calculate the portfolios mean return and variance using the following details, $R = (0.2, 1.6, 0.9)^T$, $W = (0.3, 0.4, 0.4)$ and **7**

$$CV = \begin{bmatrix} 1.12 & 1.4 & 0.9 \\ 1.4 & 2.11 & 0.60 \\ 0.9 & 0.60 & 1.32 \end{bmatrix} \text{ find } \bar{r} \text{ \& } \sigma^2 \text{ for portfolio.}$$

6. (a) Write a short note on portfolio diagram and choice of asset. **7**

(b) Consider a portfolio of three assets A, B & C with the following properties. **7**

$$\bar{r}_A = 0.12,\ \bar{r}_B = 0.41,\ \bar{r}_C = 0.16$$

$$\sigma_A = \sigma_B = \sigma_c = 1 \ \&\ \sigma_{AB} = \sigma_{BC} = \sigma_{AC} = 0$$

For fixed $\bar{r} = 0.25$ find the minimum variance portfolio.

7. Attempt any **four** of the following in short : **8**

(a) Define inflation and write its formula.

(b) Write future value of 100 after one year with annual interest rate 10%.

(c) Define MIIR.

(d) Write the Formula for Fisher Weill Duration for discrete compounding.

(e) Define diversification in portfolio.

(f) Write the statement of two fund theorem.