

## Integ M.Sc. Sem.-3 (App. Geo.) Examination

AGL 204

Mathematics

Time : 2-00 Hours]

December 2021

[Max. Marks : 50

Instructions: All questions in **Section-I** carry equal marks.

Attempt any **Three** questions in **Section- I**.

Questions in **Section -II** is **Compulsory**.

**Section-I**

Q- I	(A) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ using Comparison test.	7
	(B) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{5^{n-1}}{n!}$ Using Ratio test.	7

Q-II	(A) Test the convergence of the series $\sum \left(\frac{n+1}{3n}\right)^n$ using Root test.	7
	(B) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ using Comparison test.	7

Q-III	(A) Test the convergence of the series $u_n = \left(\frac{n}{2n+1}\right)^n$ using Root test.	7
	(B) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ Using Ratio test.	7

Q-IV	(A) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$ .	7
	(B) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$ using Root test.	7

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N-740.2

Q-V	(A) Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$ .	7
	(B) Prove that $\lim_{x \rightarrow \infty} \frac{1^2+2^2+3^2 \dots + x^2}{x^3} = \frac{1}{3}$ .	7

Q-VI	(A) Evaluate $\lim_{x \rightarrow 0} \frac{\log(1+x^3)}{\sin^3 x}$ .	7
	(B) Evaluate the limits by definition if exist: $\lim_{(x,y) \rightarrow (2,3)} [3xy]$ .	7

Q-VII	(A) Verify Cauchy's mean value theorem for the function $f(x) = \sqrt{x}$ and $g(x) = 2x + 1$ in the interval $[1,4]$ . If possible, find c.	7
	(B) Expansion of function $f(x) = \sin x, n \in \mathbb{R}$ is $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots, \forall n \in \mathbb{R}$ .	7

Q-VIII	1. Find $\lim_{x \rightarrow -1} \frac{x^2 - 5x - 6}{x^2 + 4x + 3}$ .	7
	2. Find $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x - 4}$ .	7

**Section II**

Q-QIX	1. Given $a_n = 3 + (n - 1)(9)$ , what is the common difference? (a) -1 (b) 3 (c) -9 (d) 9	1
	2. The series $\frac{1}{3^p} + \frac{1}{5^p} + \frac{1}{7^p} \dots$ is convergent if (a) $p > 1$ (b) $p = 1$ (c) $p \leq 1$ (d) $p = 0$	1
	3. The Arithmetic mean of two number 8 and 16 is (a) 10 (b) 14 (c) 8 (d) 12	1

N-740.3

4. What is the 20 <sup>th</sup> term of the sequence defined by $a_n = (n - 1)(2 - n)(3 + n)$ ? (a) 7866 (b) 8888 (c) 6677 (d) -7866	1
5. $\lim_{n \rightarrow \infty} \frac{\log n}{n} = \underline{\hspace{2cm}}$ (a) 1 (b) -1 (c) $\infty$ (d) 0	1
6. Find $\lim_{x \rightarrow 0} [5^x(x^3 + 1)]$ . (a) 1 (b) 0 (c) -1 (d) 2	1
7. $\lim_{n \rightarrow \infty} (n)^{\frac{1}{n}} = \underline{\hspace{2cm}}$ (a) 0 (b) -1 (c) 1 (d) $\infty$	1
8. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \underline{\hspace{2cm}}$ (a) $e$ (b) 0 (c) 1 (d) -1	1

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