Seat No. : _______________

MK-111

May-2022

Int. M.Sc. (CA & IT), Sem.-II

Matrix Algebra and Graph Theory

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Use of simple calculator is allowed.
(2) In **Section – I**, attempt any **three** questions out of given **five**.
(3) In **Section – II**, attempt any **eight** MCQs out of given **ten**.

SECTION – I

1. Attempt **all**. **2 × 7 = 14**

(A) State and prove hand-shaking lemma. Also, show that in any graph G there is an even number of odd vertices.

(B) Solve the following system of linear equations using Row Echelon method :

$$x - y - z = 7$$
$$2x + 3y - 4z = -2$$
$$3x - 4y + 7z = -5$$

2. Attempt **all**. **2 × 7 = 14**

(A) Find Row Echelon form of the matrix $A = \begin{pmatrix} 3 & -1 & 1 & 2 \\ 2 & 3 & 2 & 2 \\ 1 & -1 & 3 & 7 \end{pmatrix}$.

(B) Find rank of the matrix $A = \begin{pmatrix} -1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & -2 & 4 \end{pmatrix}$.

3. Attempt **all**. **2 × 7 = 14**

(A) If $A = \begin{pmatrix} 2 & -1 & -6 \\ -5 & 3 & -4 \\ -6 & 7 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 3 & 2 \\ 2 & -3 & 4 \\ 6 & -7 & 2 \end{pmatrix}$ then show that $(AB)^T = B^T A^T$.

(B) Calculate inverse of a matrix (if it exists) $A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 1 & 2 \\ 2 & 7 & 2 \end{pmatrix}$.

4. Attempt **all**. **2 × 7 = 14**

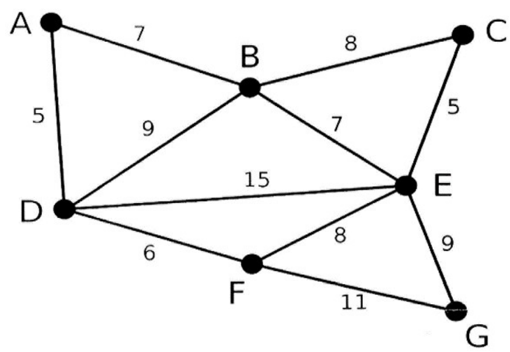
(A) Find characteristic polynomial of $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}$. Also verify Cayley-Hamilton theorem.

(B) Let $T : R^2 \to R^3$ be a mapping defined by
$T(x, y) = (y, y - x, 2x + 3y)$. Prove that T is linear.

5. Attempt **all**. **2 × 7 = 14**

(A) Check whether $A = \{(-1, 2, 3), (2, 1, -2), (1, 0, 3)\}$ is a linearly independent or linearly dependent subset of R^3 ?

(B) Apply Prims algorithm to find out minimal spanning tree. Also, calculate the minimum weight of the resultant graph.



SECTION – II

Attempt any **eight**. **1 × 8 = 8**

(1) A graph is called simple if ______.

(a) It has loop and parallel edges.
(b) It has loop but no parallel edges.
(c) It has no loop but parallel edges.
(d) It has no loop and no parallel edges.

(2) Number of edges in a complete graph is ______.

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$

(c) $\frac{(n-1)(n+1)}{2}$ (d) None

(3) The solution set of the system of linear equations $x = y$ and $x = -y$ contains ______ elements.

(a) 0 (b) 1
(c) 2 (d) Infinitely many

(4) If $A = \begin{pmatrix} 8 & 5 \\ 7 & 6 \end{pmatrix}$ then the value of det $(A^{121} - A^{120})$ is ______.

(a) 0 (b) 1

(c) 2 (d) 2/3

(5) If A is m × n matrix such that both AB and BA are defined, then B is a matrix of order _____.

(a) n × n (b) m × m

(c) n × m (d) None

(6) If $A = \begin{pmatrix} -1 & 3 & -6 \\ -5 & 3 & -1 \\ 8 & -9 & -7 \end{pmatrix}$ then trace of A is ______.

(a) 5 (b) 3

(c) –5 (d) None

(7) If a matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & x \end{pmatrix}$ is singular (non-invertible) then the value of x is _____.

(a) –2 (b) 2

(c) –1 (d) None

(8) If $A = \begin{pmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{pmatrix}$ is symmetric then $x =$ _____.

(a) 6 (b) – 6

(c) 4 (d) – 4

(9) If the order of A is 4 × 3, the order of B is 4 × 5 and the order of C is 7 × 3, then the order of a matrix $(A^TB)^TC^T$ is _____.

(a) 5 × 3 (b) 4 × 5

(c) 5 × 7 (d) 4 × 3

(10) Find the value of x + y if $\begin{pmatrix} 2x & 5 \\ 7 & -y \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 7 & 3 \end{pmatrix}$ ______.

(a) 4 (b) 1

(c) –3 (d) 6