1005E145

Candidate's Seat No : ________

MSc Sem.-3 Examination

501

Mathematics

May 2022

Time : 2-00 Hours] **[Max. Marks : 50**

Instructions:

1. Each question in **Section-I** carry equal 14 marks.
2. Attempt any **Three** questions in **Section-I**.
3. Questions in **Section-II** are **COMPULSORY**.

Section-I

1. (A) Prove that $\mathbb{R}^n$ is a vector space over $\mathbb{R}$. What is the dimension of $\mathbb{R}^n$ over $\mathbb{R}$? 7

   (B) Let N be a normed linear space. Prove the following: 7

   (i) For $x, y \in N$, $| \, \|x\| - \|y\| \, | \leq \|x - y\|$.

   (ii) the norm is a continuous function.

2. (A) Is the set $A = \{(x_1, x_2, x_3)/x_1 + x_2 + x_3 = 1\}$ a subspace of the real linear space $\mathbb{R}^3$? Justify your answer. 7

   (B) State (carefully) Zorn's lemma. Define each term used in the statement of this lemma. 7

3. (A) When we say that a linear map $T : N \to N'$ is continuous (bounded)? Give an illustration. 7

   (B) Give an example of a discontinuous linear transformation. 7

4. (A) State and prove Hahn-Banach theorem. 7

   (B) If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, prove that $(l_p^n)^* = l_q^n$. 7

5. (A) Let N be a non-zero normed linear space. If N is a Banach space, prove that $\{x : \|x\| = 1\}$ is complete. 7

   (B) For any non-empty subset S of a Hilbert space H, prove that $S^\perp$ is always a closed subspace of H. 7

6. (a) If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, prove that there exists a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$. 7

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(b) State and prove Schwartz inequality. **7**

7. (A) Let L be a non-zero finite-dimensional linear space of dimension n and W be a linear subspace of L of dimesnsion m, show that L/W is a linear space. What is the dimension of L/W? **7**

(B) Give an example of a normed linear space which is not complete. Justify your answer. **7**

8. (A) Sketch the following sets: **7**

(i) $S = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : \|x\|_2 \leq 1\}$.

(ii) $S = \{x = (x_1, x_2) \in \mathbb{R}^2 : \|x\|_\infty = 1\}$.

(iii) $S = \{x = (x_1, x_2) \in \mathbb{R}^2 : \|x\|_1 = 1\}$.

(B) State the following theorems.(Do not prove) **7**

(i) The closed graph theorem

(ii) Open mapping theorem.

(iii) Uniform boundedness theorem.

Section-II

8

(1) If M is a two-dimensional subspace of the real space $\mathbb{R}^3$, then

(A) M is a line through the origin.

(B) M is a plane through the origin.

(C) $M = \{0\}$.

(D) M is the entire space $\mathbb{R}^3$.

(2) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be that linear transformation such that $T(1,1) = (1, -2)$ and $T(1,0) = (-4, 1)$, then $T(5, -3)$ equals

(A) $(-35, 14)$ (B) $(-35, 6)$ (C) $(14, -35)$ (D) $(35, 3)$

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(3) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear mapping defined by $T(x, y, z) = (x - 3y - 2z, y - 4z, 0)$, then T is

(A) one-one

(B) onto

(C) invertible

(D) none of the above

(4) The inequality $\sum_{i=1}^{n} |x_i y_i| \leq \|x\|_p \|y\|_q$ is called

(A) Cauchy's inequality

(B) Hölder's inequality.

(C) Minkowski's inequality.

(D) None of these.

(5) If $T : N \to N'$ is linear and N is finite dimensional then..

(A) T is continuous

(B) $T = 0$

(C) Both N and N' are finite dimensional.

(D) None of these

(6) Which of the following subspaces of normed linear space l_∞ is not closed?

(A) c (B) c_0 (C) c_{00} (D) None of these

(7) For $x \in X$, the norm in the inner product space X is

(A) $\|x\| = \langle x, x \rangle$

(B) $\|x\| = \sqrt{\langle x, x \rangle}$

(C) $\|x\| = \langle x, x \rangle^2$

(D) none of the above.

(8) If N is a finite-dimensional normed linear space of dimension n, then the dimension of conjugate space N^* of N is

(A) equal to n

(B) less than n

(C) grater that n

(D) infinite