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0806E475

Candidate's Seat No :_____

M.Sc Sem.-2 Examination

P - 408

Statistics

June 2022

Time : 2-00 Hours] **[Max. Marks : 50**

Instructions: 1. All Questions in **Section-I** carry equal marks.
2. Attempt any **THREE** questions in **Section-I**
3. Question IX in **Section-II is COMPULSORY**

Section-I

Q-1(A) Define Neyman type –A distribution. Obtain its probability generating function. Derive probability mass function using probability generating function. Obtain Mean and Variance. 07

(B) Define Poisson Binomial distribution. Obtain its probability generating function. Obtain recurrence relation for the probability of this distribution. 07

Q-2(A) Let $X_1, X_2,\ldots,X_N$ are N identically independently distributed random variables and N is also a random variable independent of X_i's. If $S_N = Y = \sum_i X_i$ then show that
(i) $E(S_N) = E(N).E(X)$
(ii) $V(S_N) = E(N).V(X) + V(N).\{E(X)\}^2$ 07

(B) Define Poisson Negative Binomial distribution. Obtain recurrence relations for Probability and descending factorial moments for this distribution. 07

Q-3(A) Define non-central chi-square distribution. Obtain probability density function of non-central Chi-square distribution. 07

(B) Write applications of non-central distributions. State and prove the relations between non- central chi-square, non-central F and non-central t distributions. 07

Q-4(A) Define non-central **F** distribution with degrees of freedom n_1 and n_2. In usual notations obtain probability density function of non-central **F** distribution. 07

(B) Define non-central **t** statistic. In usual notations obtain probability density function of non-central **t** distribution. 07

Q-5(A) Define the sample range. Obtain the distribution of sample range for infinite range population. State the distribution of sample range for finite range population. 07

(B) Obtain the distribution of sample median when (i) n is odd number and (ii) n is even number. 07

Q-6 (A) Obtain the correlation coefficient between rth and sth order statistics for the uniform distribution U(0,1). Hence write the correlation coefficient between the smallest and largest order statistics. 07

(B) Explain procedure of obtaining Confidence Interval for p^{th} Quantile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a random sample of size 9 taken from any continuous distribution with cumulative distribution function $F_X(x)$ then obtain
$P(X_{(3)} < \text{Population median} < X_{(7)})$. 07

Q-7(A) In usual notations obtain the formula for the correlation coefficient between the rank-

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orders and variate values. 07

(B) Find the correlation coefficient between the rank-orders and variate values for a random sample of size N from the (i) Uniform distribution and (ii) Exponential distribution. 07

Q-8(A) Obtain the distribution of sample range for finite range population. If X has uniform distribution $U(0,\theta)$, then show that $E(R) = [(n-1)/(n+1)]\theta$ based on a random sample of size n taken from the given distribution, where R = sample range. 07

(B) Define Wilcoxon ranksum statistic. Define its distribution function. For n=2 and m=4, obtain the distribution of W. In usual notations test the hypothesis H_0: $\Delta = 0$ against H_1: $\Delta > 0$ for W=9. Write your conclusion. 07

Section II

Q-9 Choose correct answer. (Any eight) 08

1. Descending factorial cumulant generating function H(t) is defined as
   (A) Log $E(1+t)^X$
   (B) Ln $E(1+t)^X$
   (C) Exp($E(1+t)^X$)
   (D) Log $E(1-t)^{-X}$

2. If N be the number of egg masses in a certain region and $X_1, X_2, \ldots, X_n$ be the number of larvae in the N egg masses then the total number of larvae in the given region follows _____.
   (A) Poisson-Binomial distribution.
   (B) Poisson-Poisson distribution.
   (C) Poisson distribution
   (D) Posson-Negative Binomial distribution

3. If N be the number of accidents at in a certain place during a week and $X_1, X_2, \ldots, X_n$ be the number of injured persons in the N accidents respectively then the total number of injured persons at a given place during a week follows _____.
   (A) Poisson-Binomial distribution.
   (B) Poisson-Poisson distribution.
   (C) Poisson distribution
   (D) Posson-Negative Binomial distribution

4. Which one of the following statement is not true?
   (A) For Poisson Binomial distribution mean is less than variance.
   (B) For Poisson Pascal distribution mean is less than variance.
   (C) Neyman type-A distribution tends to Neyman type-B distribution.
   (D) Neyman type-B distribution tends to Neyman type-A distribution.

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5. The probability generating function of the Poisson Negative Binomial distribution is

(A) $G(Z) = e^{-\lambda - \lambda(q-pz)^{-n}}$

(B) $G(Z) = e^{\lambda - \lambda(q-pz)^{-n}}$

(C) $G(Z) = e^{-\lambda + \lambda(q-pz)^{-n}}$

(D) $G(Z) = e^{-\lambda + \lambda(q+pz)^{-n}}$

6. The recurrence relation for the probability for the Neyman type-A distribution is

(A) $p_{r+1} = \frac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-j}$

(B) $p_{r+1} = \frac{\mu_1' e^{-m}}{r-1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-j}$

(C) $p_{r+1} = \frac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \frac{m^j}{j} p_{r-j}$

(D) $p_{r+1} = \frac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-1}$

7. The probability mass function of the Poisson Bionomial distribution is

(A) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^x q^{nr-x} \frac{\lambda^r}{r!}$

(B) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^{-x} q^{nr-x} \frac{\lambda^r}{r!}$

(C) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^x q^{-nr-x} \frac{\lambda^r}{r!}$

(D) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^x q^{nr-x} \frac{\lambda^{-r}}{r!}$

8. If X is a non-central chi-square variate with degrees 5 and non-centrality parameter δ is also 5 then E(X) and V(X) are respectively

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(A) (10, 30)
(B) (15, 50)
(C) (5, 10)
(D) None of these

9. If a statistics t follows Student's t distribution with degrees of freedom n, then t^2 follows

(A) Student's t distribution with n^2 degrees of freedom
(B) Snedecor's F distribution with (1, n) degrees of freedom
(C) Snedecor's F distribution with (n, 1) degrees of freedom
(D) None of these

10. If a random variable X has a chi-square distribution with degrees of freedom r and a random variable Y has a non-central chi-square distribution with degrees of freedom 1 and non-centrality parameter λ then the distribution of the random variable Z=X+Y is

(A) Chi-square with degrees of freedom r+1
(B) Chi-square with degrees of freedom r
(C) Non-central chi-square distribution with degrees of freedom r+ 1 and non-centrality parameter λ
(D) None of these

11. Which one of the following statement is not true?

(A) When 'v=1', student's t distribution tends to Weibull distribution.
(B) When 'v=1', student's t distribution tends to Cauchy distribution.
(C) The sampling distribution of F-statistic does not involve any population parameter.
(D) The non-central Chi-square distribution is the mixture of central Chi-square distribution and Poisson distribution.

12. The distribution of smallest ordered statistic is

(A) $F_y(x) = \left(1 - F_x(x)\right)^n$
(B) $F_y(x) = \left(1 - (F_x(x))^n\right)$
(C) $F_y(x) = 1 - \left(1 - F_x(x)\right)^n$
(D) $F_y(x) = 1 - \left(1 - (F_x(x))^n\right)$

13. If X_i follows Geometric distribution with parameter p_i ($0 < p_i < 1$, $q_i = 1 - p_i$, $\forall i$) (i=1,2,....,n) and are independent then the distribution of the smallest order statistics is

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(A) Geometric with parameter $\left(1-\prod_{i=1}^{n} q_i\right)$

(B) Geometric with parameter $\left(\prod_{i=1}^{n} q_i\right)$

(C) Geometric with parameter $\left(1-\sum_{i=1}^{n} q_i\right)$

(D) Geometric with parameter $\left(\sum_{i=1}^{n} q_i\right)$

14. If $x=x_{(1)}$ and $y=x_{(n)}$ are smallest and largest order statistics respectively then their joint distribution is

(A) $f(x,y)=n(n-1)f_x(x)f_x(y)\left(F_x(x)-F_x(y)\right)^{n-2}$

(B) $f(x,y)=n(n-1)f_x(x)f_x(y)\left(F_x(x)-F_x(y)\right)^{n-1}$

(C) $f(x,y)=n(n-1)f_x(x)f_x(y)\left(F_x(y)-F_x(x)\right)^{n-1}$

(D) $f(x,y)=n(n-1)f_x(x)f_x(y)\left(F_x(y)-F_x(x)\right)^{n-2}$

15. If a random sample of size 5 is taken from Uniform distribution then the probability density function of the sample median is

(A) the probability density function of the third order statistics

(B) the probability density function of the fifth order statistics

(C) the probability density function of the first order statistics

(D) None of these

16. The mean and variance of rth ordered statistic for U (0, 1) distribution are Respectively

(A) (r/(n+2), r(n-r+1)/[(n+1)^2(n+2)])

(B) (r/(n+1), r(n-r+1)/[(n+1)^2(n+2)])

(C) (r/(n+2), r(n-r+1)/[(n+1)(n+2)^2])

(D) (s/(n+1), s(n-s+1)/[(n+1)^2(n+2)])